# Some Subsidiary Relations from the ECE Lemma

## 1) Standard Derivation (Appendix J, volume 1)

$$\square q^a_\lambda := R q^a_\lambda \quad -(1)$$

$$R := q^\lambda_a \partial^\mu \left( \Gamma^\nu_{\mu\lambda} q^a_\nu - \omega^a_{\mu b} q^b_\lambda \right). \quad -(2)$$

The ECE Lemma is a subsidiary mathematical proposition (1). The scalar curvature is defined as in eq. (2), in terms of the tetrad, gamma connection and spin connection. This means that provided R is defined as in eq. (2), the identity (1) is true. The identity (1) is sought because it has the structure of the main wave equations of physics, and so unifies quantum mechanics and general relativity.

## 2) Derivations in Paper 37

$$\square q^a_\lambda := R q^a_\lambda \quad -(3)$$

$$R := q^\lambda_a \left( \left( D^\mu \Gamma^\nu_{\mu\lambda} \right) q^a_\nu - \left( D^\mu \omega^a_{\mu b} \right) q^b_\lambda + \Gamma^\nu_{\nu\mu} \omega^{\mu a}{}_b - \Gamma^\nu_{\nu\mu} \Gamma^{\mu\nu}{}_\lambda \right). \quad -(4)$$

This is a more complicated version of R, so eq. (2) is preferred by Ockham's Razor. Eq. (3) is an identity provided that eq. (4) is true.

2)

3) Development of Eq. (2)

It is shown in paper 37 (volume 2), ~~that~~ eq. (2) may be written as:

$$R := -g^{\mu\sigma} \left( \Gamma^{\nu}_{\mu a} \omega^{a}_{\sigma\nu} + \omega^{\nu}_{\mu a} \Gamma^{a}_{\sigma\nu} \right) \quad -(5)$$

This expresses R in terms of the metric, gamma and spin connection.

4) Self Check (Paper 38)

Start with the tetrad postulate:

$$D_\mu q^a_\lambda = \partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda - \Gamma^{\nu}_{\mu\lambda} q^a_\nu = 0 \quad -(6)$$

The Lemma is the identity:

$$D^\mu \left( D_\mu q^a_\lambda \right) := 0 \quad -(7)$$

i.e.

$$D^\mu \left( \partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda - \Gamma^{\nu}_{\mu\lambda} q^a_\nu \right) := 0 \quad -(8)$$

Using the Leibnitz Theorem and the tetrad postulate again, eq. (8) is:

$$\left( D^\mu \partial_\mu \right) q^a_\lambda + \left( D^\mu \omega^a_{\mu b} \right) q^b_\lambda - \left( D^\mu \Gamma^{\nu}_{\mu\lambda} \right) q^a_\nu = 0 \quad -(9)$$

3) Now construct a self-check or cross-check by expressing eq (7) as:

$$\partial^{\mu}\left(D_{\mu} V^{a}_{\lambda}\right) = 0 \quad -(10)$$

So we know that eq (10) and eq (9) must give the same result. This is cross-checked as follows.

First use the results:

$$D_{\mu}\partial^{\mu} = \Box + \Gamma^{\mu}_{\mu\lambda}\partial^{\lambda} \quad -(11)$$

$$D^{\mu} = g^{\mu\nu} D_{\nu} \quad -(12)$$

$$\partial_{\mu} = g_{\mu\nu}\partial^{\nu} \quad -(13)$$

This means:

$$D^{\mu}\partial_{\mu} = g^{\mu\nu} D_{\nu}\, g_{\mu\nu}\,\partial^{\nu} = 4 D_{\mu}\partial^{\mu} \quad -(14)$$

Now use eq. (14) in eq. (9):

$$4\left(D_{\mu}\partial^{\mu}\right)V^{a}_{\lambda} + \left(D^{\mu}\omega^{a}_{\mu b}\right)V^{b}_{\lambda} - \left(D^{\mu}\Gamma^{\nu}_{\mu\lambda}\right)V^{a}_{\nu} = 0 \quad -(15)$$

i.e:

$$4\left(D_{\mu}\left(\partial^{\mu}V^{a}_{\lambda}\right) + \partial^{\mu}\left(D_{\mu}V^{a}_{\lambda}\right)\right) \quad -(16)$$
$$+ \left(D^{\mu}\omega^{a}_{\mu b}\right)V^{b}_{\lambda} - \left(D^{\mu}\Gamma^{\nu}_{\mu\lambda}\right)V^{a}_{\nu} = 0$$

4) By comparison of eq. (10) and (16) we know that the following must be true:

$$4 D_\mu \left( \partial^\mu q^a_\lambda \right) + D^\mu \omega^a_{\mu b} - \left( D^\mu \Gamma^\nu_{\mu\lambda} \right) q^a_\nu = 0 \quad -(17)$$

i.e.

$$D^\mu \left( \partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda - \Gamma^\nu_{\mu\lambda} q^a_\nu \right) = 0 \quad -(18)$$

which is

$$D^\mu \left( D_\mu q^a_\lambda \right) = 0 \quad -(19)$$

quod erat demonstrandum (Q.E.D.)

So this self-checks eq. (2). It checks that Cartan geometry is correct, and that this use of Cartan geometry is correct.

5) Bruhn's Version of the ECE Lemma.

Bruhn starts with the tetrad postulate:

$$\partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu - \Gamma^\sigma_{\mu\nu} q^a_\sigma = 0 \quad -(20)$$

Presumably, he now accepts the tetrad postulate after spending a long time asserting that it is somehow wrong.

5) He then operates on eq. (20) with $\partial^\mu$. So Bruhn now accepts eqn. (10), diametrically contradicting himself, but finally getting it right. So:

$$\partial^\mu \left( \partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu - \Gamma^\sigma_{\mu\nu} q^a_\sigma \right) = 0 \quad -(21)$$

Elementary differentiation gives:

$$\Box q^a_\nu + \left(\partial^\mu \omega^a_{\mu b}\right) q^b_\nu + \omega^a_{\mu b} \left(\partial^\mu q^b_\nu\right) - \left(\partial^\mu \Gamma^\sigma_{\mu\nu}\right) q^a_\sigma - \Gamma^\sigma_{\mu\nu} \left(\partial^\mu q^a_\sigma\right) = 0 \quad -(22)$$

Bruhn now uses the tetrad postulate twice more, thus contradicting himself twice more, because the tetrad postulate is now accepted by him but also simultaneously rejected by him. So he uses:

$$\partial^\mu q^b_\nu = \Gamma^{\mu\rho}{}_\nu q^b_\rho - \omega^{\mu b}{}_c q^c_\lambda \quad -(23)$$

$$\partial^\mu q^a_\sigma = \Gamma^{\mu\rho}{}_\sigma q^a_\rho - \omega^{\mu a}{}_c q^c_\sigma \quad -(24)$$

So he just uses eqns. (23) and (24) in eq. (22) to obtain:

6)

$$\Box q^a_{\ \nu} + \left(\partial^\mu \omega^a_{\ \mu b}\right) q^b_{\ \nu} - \left(\partial^\mu \Gamma^\sigma_{\ \mu\nu}\right) q^a_{\ \sigma}$$
$$+ \omega^a_{\ \mu b}\left(\Gamma^{\mu\rho}_{\ \ \ \nu} q^b_{\ \rho} - \omega^{\mu b}_{\ \ \ c} q^c_{\ \lambda}\right)$$
$$- \Gamma^\sigma_{\ \mu\nu}\left(\Gamma^{\mu\rho}_{\ \ \ \sigma} q^a_{\ \rho} - \omega^{\mu a}_{\ \ \ c} q^c_{\ \sigma}\right) = 0 \quad -(25)$$

He then re-arranges dummy indices to obtain:

$$\Box q^a_{\ \nu} + \left(\partial^\mu \omega^a_{\ \mu c} - \omega^a_{\ \mu b}\omega^{\mu b}_{\ \ \ c}\right) q^c_{\ \nu}$$
$$- \left(\partial^\mu \Gamma^\rho_{\ \mu\nu} + \Gamma^\sigma_{\ \mu\nu}\Gamma^{\mu\rho}_{\ \ \ \sigma}\right) q^a_{\ \rho}$$
$$+ 2\omega^a_{\ \mu c}\Gamma^{\mu\sigma}_{\ \ \ \nu} q^c_{\ \sigma} = 0 \quad -(26)$$

Finally he tries to create the false impression ~~that~~ eq. (2) is somehow wrong by <u>deliberately</u> <u>omitting</u> <u>the final step</u>, <u>which is</u>:

$$R := q^\nu_{\ a}\Big(\left(\partial^\mu \Gamma^\rho_{\ \mu\nu} + \Gamma^\sigma_{\ \mu\nu}\Gamma^{\mu\rho}_{\ \ \ \sigma}\right) q^a_{\ \rho}$$
$$- \left(\partial^\mu \omega^a_{\ \mu c} - \omega^a_{\ \mu b}\omega^{\mu b}_{\ \ \ c}\right) q^c_{\ \nu}$$
$$- 2\omega^a_{\ \mu c}\Gamma^{\mu\sigma}_{\ \ \ \nu} q^c_{\ \sigma}\Big) \quad -(27)$$

$$\boxed{\Box q^a_{\ \nu} := R\, q^a_{\ \nu}} \quad -(28)$$