Erratum: On the contractions of vectorial lattice representations

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In [1] it has been stated that for any finite height sublattice M of a complemented modular lattice C and surjective homomorphism $\pi : M \to M'$ there exists a "contracing" element u in C such that $x \mapsto u + \sigma x$ is a lattice embedding of M' into C where $\sigma x = \inf\{a \in M \mid \pi a = x\}$. Yet, in Case 2 of Lemma 4 it was not verified that the given chain K is admissible for all M_a, a a coatom. An example, where no such K exists, is obtained from the chain $D = \{0 < c < 1\}$ with $M = D^2$ and θ_1 the congruence generated by the quotient $(c, c)/(0, 0)$.

Moreover, we provide an example where M is even an isometric sublattice of C but no contracting u exists. The example builds on Lemma 1.3 in Pogunkte [2]. Consider a field K with at least 7 elements, $\lambda \neq 0, 1$ in K, and V a 6-dimensional K-vector space with basis $e_1, e_2, e_3, f_1, f_2, f_3$. Use $\langle \dots \rangle$ do denote the subspace generated by given vectors. Define

$$a_1 = \langle e_1, e_2, e_3 \rangle,\ a_2 = \langle f_1, f_2, f_3 \rangle,\ a_3 = \langle e_1 + f_1,\ e_2 + f_2,\ e_3 + f_3 \rangle$$

$$a_4 = \langle e_1 + \lambda f_1,\ f_1 + e_2 + \lambda f_2,\ f_2 + e_3 + \lambda f_3 \rangle, b = \langle e_1, f_1 \rangle,\ c = \langle e_1, e_2, f_1, f_2 \rangle$$

Then the sublattice L generated by these elements in the lattice C of all vector subspaces of V is a subdirect product of 3 copies of the 6-element height 2 lattice M_4, the subdirect decomposition given by the two neutral elements b and c. Choose an element $b_i \notin L$ in each interval $[a_i c,\ a_i + b]$ of C, and two elements $c_1, c_2 \notin L$ in the interval $[c, 1]$ of C - 1 the top element of C. Addding these elements to L one obtains a sublattice M of C where these are doubly irreducible. Let θ be the congruence of M generated by the quotient c/b and $\pi : M \to M' = M/\theta$ the canonical projection. Observe that the non-singleton classes of θ are the interval $[b, c]$ of M and the $\{a_i c,\ a_i b\}$ and $\{a_i + c,\ a_i + b\}$. In particular, we have 1 and the a_i, b_i, c_i fixed under $\sigma\pi$.

Now, assume there were a contracting u in C. Then $u + c_i = u + \sigma\pi c_i < 1 = u + \sigma\pi 1$, hence $u \leq c_i$ and $u \leq c = c_1 c_2$. Also, $u + b = u + \sigma\pi b$ has to be of co-dimension 2 whence $u + b = c$. On the other hand, $x \mapsto x + u$ has to be an embedding of the interval $[0, b]$ of M into the interval $[u, c]$ of C. By modularity, it follows that $ub = 0$. Now, as observed by Pogunkte, the quadruple $a_i c (i \leq 4)$ of subspaces of the vector space c is indecomposable (since it describes an indecompsble automorphism) and this is expressed by the fact that $a_i b_i + u = c$ for at least one i (taking Poguntke's argument, dually). But then for this i one has $\sigma\pi a_i + u = a_i + u = c_i + u = \sigma\pi c_i + u$, a contradction.

References

[1] C. Herrmann, On the contractions of vectorial lattice representations, *Order* **8** (1991), 175–281

[2] W.Poguntke, Zerlegung von S-Verbänden, Math. Z. **142** (1975), 47–65

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