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★**On quadruples of normal subgroups.**

Proceedings of the Lattice Theory Conference (Ulm, 1975), pp. 92–99. *Univ. Ulm, Ulm,* 1975.

From the author's introduction: "I. M. Gel′fand and V. A. Ponomarev [*Hilbert space operators and operator algebra* (Proc. Internat. Conf., Tihany, 1970), pp. 163–237, North-Holland, Amsterdam, 1972; MR0357428] classified the directly indecomposable finite dimensional vectorspaces with a quadruple of vector subspaces over an algebraically closed field. Although no similar classification is possible for groups with a quadruple of normal subgroups, the sublattices generated by quadruples in the lattice of all vector subspaces [all normal subgroups] are quite similar in structure. More precisely, in both cases any such lattice is a subdirect product (given by neutral elements) of a lattice the factors of which can be presented by diagrams and several lattices which are generated by normalized frames such that for any of them the generating quadruple is related to the frame according to a suitable system (of non zero defect) in the Gelfand-Ponomarev classification. Since the frame generated lattices have been determined in [the work by the autor and A. Huhn [*Lattice theory* (Proc. Colloq., Szeged, 1974), pp. 97–136, North-Holland, Amsterdam, 1976] we get a complete list of four-generated subdirectly irreducible normal lattices, i.e., lattices satisfying all lattice theoretic identities which are valid in all lattices of normal subgroups."

{For the entire collection see MR0419305.}

{For the collection containing this paper see MR0419305}

Contradicting the above claim, from Lemma 8 in Math.Ann.265, 513-527 it follows that the subgroup lattice of an abelian group of type p^2,p^2,p^2,p (which does not admit any spanning frame) is generated by 4 elements. The claim remains valid for "elementary arguesian" lattices, a class extending the variety generated by subspace lattices of vector spaces only very slightly (Algebra Universalis 18, 225- 259