Two-phase flow in porous media are described by the Muskat problem, which is a moving boundary problem purposed originally to describe the interaction of water and oil in an oil sand.
By establishing the parabolic character of the mathematical setting, we prove that the problem is well-posed and determine the stability properties of the steady-state solutions.
When the fluids form thin layers, the problem may be approximated by a strongly coupled degenerate parabolic system.
The system has only the fluid interfaces as unknowns and, when neglecting the surface tension effects, it appears  as the two-phase generalization of the Porous Medium Equation.
The global existence  of non-negative weak solutions and their asymptotic properties will be discussed in the situation when the equations are defined on a bounded interval or on the whole real line.
The talk is based on joint works with J. Escher, Ph. Laurencot, and A.-V.