FUNCTIONAL CALCULUS FOR SEMIGROUPS USING TRANSFERENCE METHODS

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Abstract. In this talk some recent results about functional calculus for semigroup generators will be explained.

Let $-A$ generate a C_0-semigroup on a Banach space X. *Functional calculus* theory provides a method of defining, for f a holomorphic function on a right half-plane, an operator $f(A)$ on X. It then aims to study the correspondence $f \mapsto f(A)$ and to relate properties of $f(A)$ to those of f. In many settings norm bounds for $f(A)$ are important, for instance in the theory of evolution equations.

The definition of $f(A)$ is first made for a class of elementary functions, the Laplace transforms of measures of bounded variation. It is then extended to a larger class by so-called regularization. A *transference method* is a means of factorizing the elementary operators $f(A)$ via convolution operators on suitable function spaces. The theory of Fourier multipliers is then used to derive norm bounds for the elementary operators, and one approximates general functions by elementary ones to extend the norm bounds.

Using such transference methods, we show that each semigroup generator on a Hilbert space has a bounded calculus for the ideal of bounded holomorphic functions f that satisfy $|f(z)| = O(e^{-\tau\,\mathrm{Re}(z)})$ as $|z| \to \infty$ for some $\tau > 0$. Moreover, the bound in this calculus grows at most logarithmically as $\tau \downarrow 0$. As a consequence, $f(A)$ is a bounded operator for each function f on a half-plane with polynomial decay at infinity. Then we show that A has an m-bounded calculus for any $m \in \mathbb{N}$, and that this property characterizes semigroup generators. Similar results hold on UMD spaces.

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