Stabilized finite element methods for convection-diffusion-reaction equations

This presentation is devoted to the numerical solution of steady scalar convection-diffusion-reaction equations by means of the finite element method. If convection dominates diffusion, the solution of the continuous problem typically possesses interior and boundary layers which cannot be resolved properly unless the mesh is extremely fine. This often leads to spurious oscillations in the numerical solution. In particular, the solutions of the classical Galerkin finite element discretization are typically globally polluted by spurious oscillations. Therefore, various stabilization techniques have been developed during the last four decades to remove or, at least, to diminish these oscillations. We shall review some of these approaches and compare them by means of numerical tests.
We shall also discuss the choice of stabilization parameters which may significantly influence the accuracy of the discrete solution. In particular, we shall show how the stabilization parameters can be computed a posteriori by means of the minimization of an appropriate target functional.