Abstract:

Many boundary value problems for semilinear elliptic partial differential equations allow very stable numerical computations of approximate solutions, but are still lacking analytical existence proofs. We propose a method which exploits the knowledge of a "good" numerical approximate solution, in order to provide a rigorous proof of existence of an exact solution close to the approximate one. This goal is achieved by a fixed-point argument (similar to the Newton-Cantorovich theorem) which takes all numerical errors into account, and thus gives a mathematical proof which is not "worse" than any purely analytical one. The method is used to prove existence and multiplicity statements for some specific examples, including cases where purely analytical methods had not been successful.