Elliptic boundary value problems on cylindrical domains

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based on joint work with Pascal Auscher

Abstract

We consider elliptic t-independent second order equations

$$-\nabla_{t,x} \cdot A(x) \nabla_{t,x} U(t,x) = 0$$

with bounded, measurable coefficients posed on a cylindrical domain $\mathbb{R}_+ \times \Omega$ with a bounded base $\Omega \subseteq \mathbb{R}^d$ and complemented with mixed Dirichlet/Neumann conditions

$$\begin{aligned} U &= 0 && (\text{on } \mathbb{R}_+ \times D) \\ \nu \cdot A \nabla_{t,x} U &= 0 && (\text{on } \mathbb{R}_+ \times (\partial\Omega \setminus D)) \end{aligned}$$

on the lateral boundary. We will classify all weak solutions to these equations for which either the non-tangential maximal function of $\nabla_{t,x} U$ or a square function associated with $\nabla_{t,x} U$ is under L^2 control on the whole cylinder. Surprisingly, this can be done independently of any well-posedness issues.