

Polymeric fluids

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The essence of modelling polymeric fluids is encapsulated in the coupling of equations describing the evolution of macroscopic quantities (like velocity, pressure and eventually also density and temperature) with an additional equation describing the microscopic structure. The influence of the processes of polymerization and fragmentation will be accounted through the dependence of viscosity on the level of polymerization and/or appearance of the extra stress tensor. We concentrate on a dilute solution of polymer chains suspended in a non-Newtonian solvent, and we assumed that individual polymer chains do not interact with one another, but can be convected by the macroscopic velocity field, and are also subject to polymerization and fragmentation processes. The problem was to find a macroscopic velocity $v(t, x) : (0, T) \times \Omega \rightarrow \mathbb{R}^3$, a pressure $q(t, x) : (0, T) \times \Omega \rightarrow \mathbb{R}_+$ and a density of the polymer molecules of the length z namely $\mu(t, x, z) : (0, T) \times \Omega \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ satisfying the following system of equations

$$\begin{aligned} v_t + v \nabla_x v + \nabla_x q - \operatorname{div}_x T &= f \\ \operatorname{div}_x v &= 0, \end{aligned} \quad (1)$$

where the stress tensor T is given by the formula

$$T(\mu, D_x v(t, x)) = \nu(\mu, |D_x v|) D_x v(t, x). \quad (2)$$

The quantity μ satisfies the following size-structure equation

$$\partial_t \mu + \operatorname{div}_x (v \mu) = \partial_z (\tau \mu) - B \mu + 2 \int_z^\infty B(y) b(z, y) \mu(t, x, y) dy. \quad (3)$$

where $B(z) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is the rate of fragmentation of polymer molecules, $b(z, y) : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow [0, 1]$ is the probability that a given particle of length y breaks into a particle of length z and $y - z$, and $\tau(z) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is the rate of coagulation of monomers. The system is complemented with boundary and initial conditions.

We shall describe different approaches to modelling of polymeric flows including FENE model, Doi model (both of them related with coupled Navier-Stokes-Fokker-Planck system) and the models describing behaviour of free monomers and polymers.

The talk is based on common results with Piotr Gwiazda (University of Warsaw) and Endre Süli (University of Oxford).