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Gradient estimates for the Stokes resolvent in bounded Lipschitz domains

Abstract: In a bounded Lipschitz domain $\Omega \subset \mathbb{R}^d$, $d \geq 3$, we consider the Stokes resolvent problem

$$\text{(SRP)} \quad \begin{cases} \lambda u(x) - \Delta u(x) + \nabla \pi(x) = f(x) & (x \in \Omega) \\ \operatorname{div}(u(x)) = 0 & (x \in \Omega) \\ u(x) = 0 & (x \in \partial\Omega) \end{cases}$$

for $\lambda \notin \mathbb{R}_-$. We will sketch the proof of the following fact:

There exists $\varepsilon > 0$ such that for all $2 \leq p < \frac{2d}{d-1} + \varepsilon$ there exists a constant $C > 0$, independent of λ, such that for all $f \in \mathrm{L}^p_\sigma(\Omega; \mathbb{C}^d)$ the weak solution (u, π) of (SRP) satisfies

$$\|\nabla u\|_{\mathrm{L}^p} + \|\pi\|_{\mathrm{L}^p} \leq C|\lambda|^{-\frac{1}{2}}\|f\|_{\mathrm{L}^p}.$$