Abstract-Soga

Abstract: Consider hyperbolic scalar conservation laws (CL) and the corresponding Hamilton-Jacobi equations (HJ) with a flux function of the form $H(x,t,p)$, where H is 1-periodic in both x and t. (CL) and (HJ) appear not only in continuum mechanics, but also in Hamiltonian/Lagrangian dynamics generated by H. It is well-known that uniqueness of x,t-periodic entropy solution of (CL) and viscosity solution of (HJ) does not hold in general, whereas uniqueness holds in approximated problems, i.e., viscous (CL)/(HJ) and discount (CL)/(HJ). Then which periodic solution is selected in the limit process of approximation? This is the selection problem. A selection criterion is made clear in viscous approximation [1] and in discount approximation [2], where one can see qualitative difference between the two approximation methods. Dynamical systems corresponding to (CL)/(HJ) and discount (CL)/(HJ), and stochastic processes corresponding to viscous (CL)/(HJ), play important roles in the arguments.

In this talk, we will discuss a similar problem with finite difference approximation of (CL)/(HJ) [3], [4], [5], demonstrating a stochastic and variational approach to finite difference approximation and showing a selection criterion which is different from the ones given in [1] and [2]. The essential technical aspect of the whole argument is to characterize the numerical viscosity by stochastic processes, which is reminiscent of the well-known stochastic approaches to parabolic/elliptic PDEs.

[1] U. Bessi, Comm. Math. Phys. 235 (2003), 495-511.
[2] A. Davini, A. Fathi, R. Iturriaga and M. Zavidovique, preprint (arXiv:1408.6712).
[3] K. Soga, Math. Comp. 84 (2015) No. 292, 629-651.
[4] K. Soga, Math. Comp. (to appear) (arXiv: 1210.2178).
[5] K. Soga, preprint (arXiv:1501.03594)