"We present joint work with Lorenzo Giacomelli and Felix Otto, in which we investigate traveling-wave solutions to a degenerate-parabolic fourth-order equation modeling the film height of a viscous thin film. The problem has a free boundary, the contact line where liquid, air, and solid meet, and we are interested in the asymptotics of solutions at the free boundary and in the interior of the droplet. Since the mobility, entering the equation, is a sum of two powers with differing exponents, one of the powers dominates the asymptotics in the respective regime. Our approach is based on a geometric dynamical systems argument, in which we construct solution manifolds at the contact line and in the interior of the droplet that are matched in three-dimensional phase space by a transversality argument. Thus we are able to rigorously recover an asymptotic solution found by L H Tanner in 1979 experimentally and later on discussed in the matched asymptotics literature. We prove that by changing the liquid-solid interactions in our model, this asymtotic law is only changed by rescaling with a length scale (being a smooth function of liquid-solid interactions) and a higher-order term. This indicates that, within our model and in line with the common philosophy, the precise modeling of liquid-solid interactions has no significant effect on the macroscopic flow properties of the thin film."