In this talk, I will discuss a new approach to continuity equations with rough coefficients. Opposed to DiPerna's and Lions's theory of renormalized solutions, the new approach is based on stability estimates
and yields the control of a logarithmic distance between particle trajectories by the velocity gradient. I will present two applications: 1) An upper bound on the order of convergence of the numerical upwind scheme in the case of Sobolev coefficients. 2) A lower bound on the rate by which two fluids can be mixed by stirring. This is partially joint work with André Schlichting (U Bonn).