The Initial Value Problem for Quasilinear SPDEs

Jonas Sauer*

We consider the variable-coefficient uniformly parabolic partial differential equation in one space dimension

$$\begin{cases} \partial_t u - a(u)\partial_x^2 u = f, \\ \qquad\qquad u|_{t=0} = u_0, \end{cases}$$

where f is a distribution which is controlled only in the low regularity norm of $C^{\alpha-2}$ for $\alpha \in (\frac{2}{3}, 1)$ in the parabolic Hölder scale, and the initial value u_0 is α-Hölder continuous. One should think of f as a random forcing. However, our analysis is mainly deterministic. In fact, we aim to obtain a C^α-estimate for u. For this purpose, one first needs to give a meaning to products of the form $u\partial_x^2 u$: since $\alpha < 1$ and since we expect $u \in C^\alpha$, there is no straight-forward interpretation of such products. Therefore, we add to the system an off-line meaning of products of the form $v\partial_x^2 v$, where v is the solution to the corresponding constant-coefficient problem. Tied to this off-line interpretation, we can also make sense of $u\partial_x^2 u$ by using an adaptation of the theory of (controlled) rough paths and show corresponding *a priori* estimates, which ultimately gives rise to (short-time) existence, uniqueness and stability with respect to $(f, u_0, v\partial_x^2 v)$. The only stochastic ingredient is that one can actually give bounds for the required off-line products in case of a stochastic forcing f.

The talk is based on an ongoing joint work with Felix Otto and Hendrik Weber.

*Max-Planck-Institut für Mathematik in den Wissenschaften, Inselstraße 22, 04103 Leipzig, sauer@mis.mpg.de