Maximal regularity of cylindrical parameter-elliptic boundary value problems

Tobias Nau

For parameter-elliptic boundary value problems in domains $V \subset \mathbb{R}^k$ with compact boundary, $\mathcal{R}$-sectoriality of the related L^p-realizations is known. We make use of this result to show $\mathcal{R}$-sectoriality for the L^p-realizations of a class of boundary value problems in unbounded cylindrical domains $\mathbb{R}^n \times V$. Due to a result of Weis, this gives maximal regularity for the corresponding Cauchy problem. The differential operators A under consideration are assumed to resolve into two parts $A = A_1 + A_2$, both parameter-elliptic, such that A_1 acts merely on $\mathbb{R}^n$ and A_2 acts merely on V. As a strong tool to treat model problems of this kind, we make use of an n-dimensional operator-valued Fourier multiplier theorem due to Haller, Heck and Noll. In this context, $\mathcal{R}$-boundedness of operator families, namely the range of $(\lambda + a_1(\cdot) + A_2)^{-1}$ plays a major role. This indicates how spectral properties of A_2 can be used to derive according properties for A. To some extent this approach allows to treat vector-valued boundary value problems in unbounded cylindrical domains.

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