Existence of steady water waves with multiple critical layers

We construct steady and periodic gravity water waves with multiple critical layers. The mathematical setting is that of the two-dimensional Euler equations with a free boundary. In the steady frame, this is an elliptic problem in an unknown domain, and the goal is to find solutions via bifurcation from a solution curve of nontrivial solutions, corresponding to rotational running streams. Using the Lyapunov—Schmidt reduction we find solutions at a particular class of eigenvalues, some of which are not simple. Those correspond to i) waves with arbitrarily many critical layers and a single crest in each period, and ii) waves with several crests and troughs in each period. The talk is based on joint work with J. Escher, G. Villari and E. Wahlén.