The talk is a kind of introduction to some fundamental questions arising in the analysis of the MHD system for incompressible fluids.

We start from the modeling of a concrete MHD problem that occurs in industrial crystal growth. In this problem, the electromagnetic fields are not confined to the region occupied by the liquid: they rather extend to a domain composed of several different materials, and satisfy discontinuous transmission conditions at their nonsmooth interfaces. The analysis of the Maxwell equations in this setting is not to carry out in the usual Sobolev space, but in functional spaces associated with the generalized operators curl and div.

We first prove an existence lemma for stationary distributional solutions. We then discuss sufficient assumptions on the data (geometry, coefficients) to prove the existence of the usual weak solution in the Navier-Stokes equations. Cornerstone for this result are the higher-integrability of the magnetic field and of its curl, that rely on recent advances in regularity theory for elliptic transmission problems. As an outlook and conclusion, we discuss some results for the case that the MHD system is also coupled to temperature.