Optimization of wear related material costs of a hydrostatic transmission system via MINLP

Lena C. Altherr[1], Philipp Leise[2], Marc E. Pfetsch[3], and Andreas Schmitt[3]

[1] Department of Electrical Engineering and Information Technology, Aachen University of Applied Sciences, Germany; altherr@fh-aachen.de,
[2] Department of Mechanical Engineering, TU Darmstadt, Germany
philipp.leise@fst.tu-darmstadt.de
[3] Department of Mathematics, TU Darmstadt, Germany
{pfetsch, aschmitt}@mathematik.tu-darmstadt.de

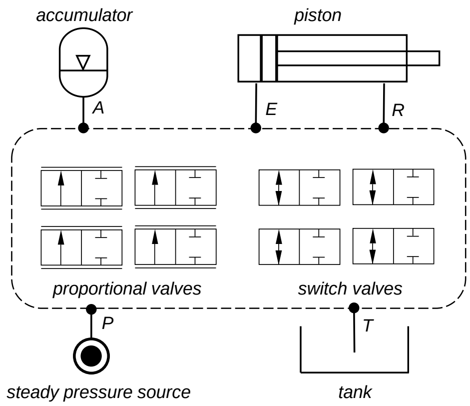
Abstract. This contribution presents a method to find an optimal topology and control for a hydrostatic transmission system that is equipped with a hydraulic accumulator. The goal is to minimize wear in the system while fulfilling a predefined load cycle given by speed and force requirements during the retraction and extension of a piston. The degrees of freedom of the design are the selection and the connection of valves with the system's piston, pressure source and tank as well as the sizing of the accumulator. We derive a mixed-integer nonlinear program, which contains continuous variables for the quasi-stationary flow, pressure and valve conditions, as well as binary variables to include selection decisions and valve circuits. Pressure and wear conditions are modeled by nonconvex nonlinear functions. To solve the problem, we use a reformulation which approximates the valve wear by a quadratic polynomial depending on volume flow and pressure difference and use a technique based on perspective cuts. Our optimization results show that the inclusion of the accumulator reduces the wear related material costs by one third.

Keywords: MINLP, Perspective Cuts, Engineering Optimization

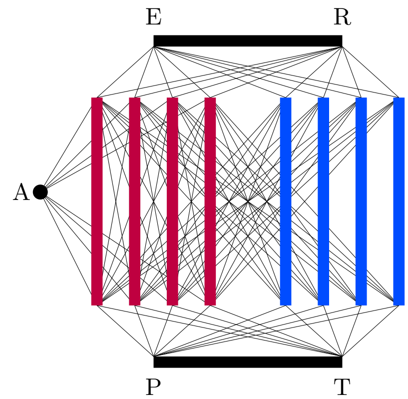
1 Introduction

Hydraulic systems are required in a multitude of technical systems, e.g., in construction and agricultural machinery. An important factor influencing the availability of such systems is component wear. This motivates the search for systematic design methods within the engineering design process that lead to a reduced component wear. We present a system synthesis approach of a hydrostatic transmission system equipped with a hydraulic accumulator using a mixed-integer nonlinear program (MINLP). In more detail, we optimize the connection of valves, the sizing of an accumulator and the control of the valves such that a given movement pattern of a piston can be realized under minimal material wear, see Fig. 1a for a schematic depiction of the technical system.

Our model is an extension of the work [1], which considers this system without an accumulator. The presented system design approach can also be used to derive



(a) Construction kit of the design problem to find an optimal connection of proportional and switch valves.



(b) Graph depicting the problem. Red and blue bars depict potential proportional and switch valves, respectively.

Fig. 1: Illustration of the problem.

optimized digital hydraulic ([6]) systems. The integration of an accumulator has multiple benefits for the system: for instance, as shown by [8, p. 249], the volume flow demand can still be satisfied while having fluctuating requirements and pressure peaks can be reduced.

In previous work [1], the complex nonlinear physics and wear constraints necessitated the usage of linearization. In contrast, here we propose a reformulation of these nonlinearities, which allows the solving of a more precise MINLP-formulation. In the following we first introduce an optimization model for the problem. Afterwards we present a reformulation and apply perspective cuts. We then conclude with a presentation of the optimal solution for some test data.

2 Optimization Model

The function of the system is given by a load cycle $\mathcal{L} \coloneqq \{\mathrm{in}, \mathrm{out}\}$ which controls the velocity $v^{\mathrm{in}}/v^{\mathrm{out}}$, force $F^{\mathrm{in}}/F^{\mathrm{out}}$ and time $t^{\mathrm{in}}/t^{\mathrm{out}}$ of retracting and extending the piston given by the points E and R in Fig. 1a. The needed pressure difference for this movement can be generated by connecting these two points with the pressure source P and/or the accumulator A via switch and proportional valves as well as by the adjustment of the proportional valves' lift.

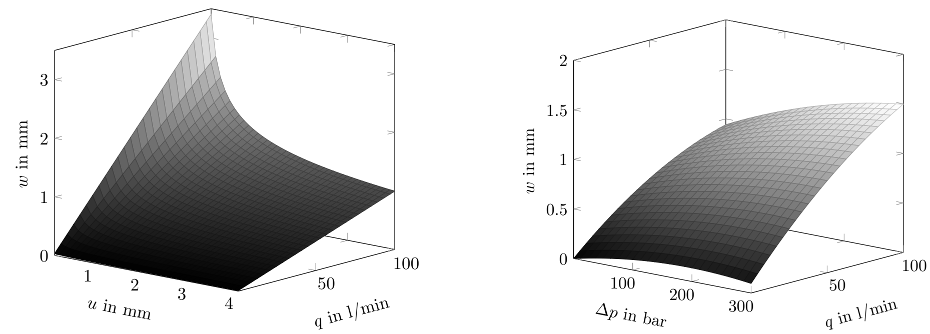
The possible designs of the hydrostatic transmission system are represented by the directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ depicted in Fig. 1b. The nodes $\mathcal{V}$ include the nodes for the extension (E) and the retraction (R) input of the piston, the accumulator (A), the pump (P) and the tank (T). Furthermore, there are two nodes for each of the eight valves. The set of arcs $\mathcal{A}$ contains an arc for each valve, where the arcs for proportional and switch valves are collected in the subsets $\mathcal{A}^{\mathrm{p}}$

$$
\begin{aligned}
\min \quad & \sum_{a \in \mathcal{A}^{\mathrm{s}}} C^{\mathrm{s}}\, x_a + \sum_{a \in \mathcal{A}^{\mathrm{p}}} C^{\mathrm{p}}\, x_a + \sum_{a \in \mathcal{A}^{\mathrm{p}}} C^{\mathrm{p}} \left\lfloor N / \lfloor \overline{w} / (w_a^{\mathrm{in}} + w_a^{\mathrm{out}}) \rfloor \right\rfloor && && \text{(1a)}\\
\text{s.t.} \quad & \sum_{a \in \delta^-(v)} q_a^\ell - \sum_{a \in \delta^+(v)} q_a^\ell = 0, && v \in \mathcal{V} \setminus \{T, A\}, \ell \in \mathcal{L}, && \text{(1b)}\\
& q_a^\ell\, (1 - x_a) = 0, && a \in \mathcal{A}^{\mathrm{c}}, \ell \in \mathcal{L}, && \text{(1c)}\\
& q_a^\ell\, (1 - y_a^\ell) = 0, && a \in \mathcal{A}^{\mathrm{p}} \cup \mathcal{A}^{\mathrm{s}}, \ell \in \mathcal{L}, && \text{(1d)}\\
& (p_u^\ell - p_v^\ell)\, x_a = 0, && a = (u,v) \in \mathcal{A}^{\mathrm{c}}, \ell \in \mathcal{L}, && \text{(1e)}\\
& (p_u^\ell - p_v^\ell)\, y_a^\ell = 0, && a = (u,v) \in \mathcal{A}^{\mathrm{s}}, \ell \in \mathcal{L}, && \text{(1f)}\\
& (p_u^\ell - p_v^\ell)\, y_a^\ell = \Delta p_a^\ell, && a = (u,v) \in \mathcal{A}^{\mathrm{p}}, \ell \in \mathcal{L}, && \text{(1g)}\\
& \rho\, q_a^\ell |q_a^\ell| = 2 \Delta p_a^\ell\, (\zeta\, d\, u_a^\ell)^2, && a \in \mathcal{A}^{\mathrm{p}}, \ell \in \mathcal{L}, && \text{(1h)}\\
& w_a^\ell \geq K(u_a^\ell, q^\ell), && a \in \mathcal{A}^{\mathrm{p}}, \ell \in \mathcal{L}, && \text{(1i)}\\
& \underline{u}\, y_a^\ell \leq u_a^\ell \leq y_a^\ell, && a \in \mathcal{A}^{\mathrm{p}}, \ell \in \mathcal{L}, && \text{(1j)}\\
& 0 \leq V^\ell \leq V^0, && \ell \in \mathcal{L}, && \text{(1k)}\\
& V^\ell = V^{\ell-1} + \sum_{a \in \delta^-(v)} q_a^{\ell-1} - \sum_{a \in \delta^+(v)} q_a^{\ell-1}, && \ell \in \mathcal{L}, && \text{(1l)}\\
& p_{A,\ell} = p^0 \left(\tfrac{V^0}{V^\ell} \right)^n, && \ell \in \mathcal{L}, && \text{(1m)}\\
& (p_E^\ell - p_R^\ell) = \tfrac{F^\ell}{A^{\mathrm{pist}}},\; q_{(E,R)}^\ell = v^\ell\, A^{\mathrm{pist}}, && \ell \in \mathcal{L}, && \text{(1n)}\\
& p_T^\ell = 1,\; p_P^\ell = \Delta P + 1, && \ell \in \mathcal{L}, && \text{(1o)}\\
& q \in \mathbb{R}^{\mathcal{A} \times \mathcal{L}},\; p, \Delta p, w \in \mathbb{R}_+^{\mathcal{A}^{\mathrm{p}} \times \mathcal{L}},\; u \in [0,1]^{\mathcal{A}^{\mathrm{p}} \times \mathcal{L}},\\
& p^0, V^0, V^{\mathrm{in}}, V^{\mathrm{out}} \in \mathbb{R}_+,\; x \in \{0,1\}^{\mathcal{A}^{\mathrm{c}} \cup \mathcal{A}^{\mathrm{p}} \cup \mathcal{A}^{\mathrm{s}}},\; y \in \{0,1\}^{(\mathcal{A}^{\mathrm{p}} \cup \mathcal{A}^{\mathrm{s}}) \times \mathcal{L}}.
\end{aligned}
$$

and $\mathcal{A}^{\mathrm{s}}$, respectively. There are two arcs for the piston and the connection of the pressure source and the tank, respectively. Lastly, there are connections between all the components with some restrictions, e.g., the accumulator can only be connected to a proportional valve and switch valves can not be connected in series. These arcs are collected in the set $\mathcal{A}^{\mathrm{c}} \subset \mathcal{A}$.

The MINLP uses the following variables: Binary variables x_a for $a \in \mathcal{A}$ signify whether an arc/connection is used in the solution. The binary variables y_a^ℓ specify whether a valve on the arc $a \in \mathcal{A}^{\mathrm{p}} \cup \mathcal{A}^{\mathrm{c}}$ is open or closed in load case $\ell \in \mathcal{L}$. To model quasi-stationary physical conditions for each load case $\ell \in \mathcal{L}$, the volume flow on each arc $a \in \mathcal{A}$ is given by q_a^ℓ. Furthermore, the pressure at a given node $v \in \mathcal{V}$ is given by p_v^ℓ. For each proportional valve $a \in \mathcal{A}^{\mathrm{p}}$ we have its valve lift u_a^ℓ, its accumulated wear w_a^ℓ and the pressure loss Δp_a^ℓ. Lastly, we model the hydraulic accumulator using preloading pressure p^0 and volume V^0 as well as the fluid volume V^ℓ.

The whole model is given by Equations (1a)–(1o). The objective is to minimize the material costs given by the usage of the valves, weighted by $C^{\mathrm{s}}/C^{\mathrm{p}}$ for the respective type and the replacement of worn out proportional valves. We



(a) According to the function K and [7].

(b) Using the quadratic polynomial f.

Fig. 2: Approximation of the wear w in terms of the valve lift u and volume flow q or pressure difference Δp and volume flow q.

use N maintenance intervals in which valves can be replaced. A valve is worn out, if the wear exceeds the upper wear bound $\overline{w}$. The number of replacements is given by $\lfloor \overline{w}/(w_a^{\text{in}}+w_a^{\text{out}}) \rfloor$. The accumulator is not considered within the objective. Constraints (1b)–(1g) enforce volume flow balance and pressure propagation between built/active components. Here, $\delta^-(v)$ and $\delta^+(v)$ denote the incoming and outgoing arcs of node v, respectively. Pressure loss only occurs for proportional valves and is approximated by Constraints (1h) following [7]. Parameters are the diameter d, the oil density ρ and the pressure loss coefficient ζ. The wear of a used proportional valve is modeled by (1i). Here, $K(u_a^\ell, q_a^\ell)$ is a nonlinear function depending on the lift and the volume flow of the considered valve and parameters like the maximum valve lift, the movement time and further aspects. This wear model was derived by a dimensional analysis using experimental data, see [7] and is depicted in Fig. 2a. We also bound the valve lift in (1j). Constraints (1k)–(1m) cover boundary conditions, volume flow balance and the pressure of the accumulator. The expression $\ell - 1$ is an abbreviation for the opposite load case of ℓ. The final constraints give boundary conditions on several nodes and arcs in the graph. This includes the necessary pressure difference and volume flow for the piston movement (A^{pist} is the area of the piston) as well as the ambient pressure of 1 bar at the tank and the constant pressure increase due to the pump of ΔP.

3 Solution Approach

To optimize the above MINLP using the solver SCIP [5], we use a reformulation and separate tight valid inequalities based on perspective cuts, see [4].

We first reformulate the wear part in the objective function, which forms a piecewise linear function with arguments $w_a^{\text{in}} + w_a^{\text{out}}$, as an aggregation of binary variables. Furthermore, we compute bounds on the volume flow and pressure variables to reformulate the bilinear constraints (1c)–(1g) with binary variables as big-M constraints. To simplify the absolute value in Constraint (1h) we split the volume flow and pressure increase variables into positive and negative parts and add binary activation variables.

Preliminary tests showed computational difficulties with the optimization of the interaction between volume flow, pressure difference, lift and wear of the proportional valves. To handle this, we neglect the valve lift variable and approximate the wear of a valve only in terms of volume flow and pressure difference using a two-dimensional quadratic polynomial f. This function is depicted in Fig. 2b. Thus, for nonnegative q_a^ℓ and Δp_a^ℓ we replace Constraints (1h)–(1j) by the system

$$2\Delta p_a^\ell\,(\zeta\, d\, \underline{u})^2 \le \rho\,(q_a^\ell)^2 \le 2\Delta p_a^\ell\,(\zeta\, d)^2, \qquad a \in \mathcal{A}^{\mathrm{p}}, \ell \in \mathcal{L}, \tag{2a}$$

$$w_a^\ell \ge f(q_a^\ell, \Delta p_a^\ell), \qquad a \in \mathcal{A}^{\mathrm{p}}, \ell \in \mathcal{L}. \tag{2b}$$

To speed up the solution process, an adaptation of perspective cuts is used. These cuts use the model structure that a binary variable switches a continuous variable on/off and that there exists a convex nonlinear relationship between the continuous variables:

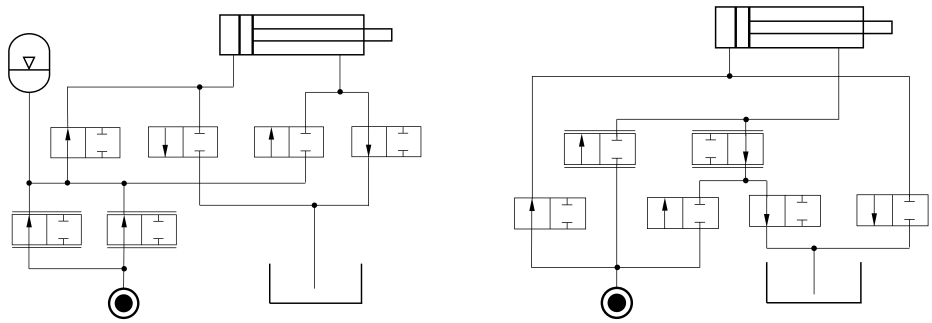
$$\{(\alpha, \beta, \gamma) \in \{0,1\} \times \mathbb{R}^n \times \mathbb{R} \,:\, \gamma \ge f(\beta), \underline{\beta}\,\alpha \le \beta \le \overline{\beta}\alpha\}.$$

This structure is also given in our problem when linking wear (γ) with volume flow and pressure loss (β) depending on activation of the valve (α). The quadratic approximation f is non-convex. However, the fixed convexity behavior of quadratic functions makes it possible to generate linear underestimators $f(\beta) \ge a^\top \beta + b$ following [2]. In [3] it is shown, that the inequalities $\gamma \ge a^\top \beta + b\ \alpha$ describe the convex hull of the above set. We dynamically separate these valid cutting planes within our optimization algorithm and significantly reduce the solution time.

4 Example Design

Exemplified results for system designs with and without an accumulator and with different valves and connections to ensure a given load-scenario are presented in Fig. 3. The pictures show the valve configurations for the extension phase, and the retraction scenario is obtained by opening closed valves and closing open valves. Here, proportional valves are considered to be twice as expensive as switching valves. The load cases are determined by $v^{\mathrm{in}} = 0.15\,\mathrm{m/s}$, $F^{\mathrm{in}} = 20\,\mathrm{kN}$, $t^{\mathrm{in}} = 20\,\mathrm{s}$, $v^{\mathrm{out}} = 0.6\,\mathrm{m/s}$, $F^{\mathrm{out}} = 5\,\mathrm{kN}$, and $t^{\mathrm{out}} = 5\,\mathrm{s}$.

The use of the hydraulic accumulator reduces the wear and thus the material costs based on the underlying model assumptions by one third. The optimized solution configures the accumulator in such a way that it can handle the retraction cycle without the pump.



(a) With accumulator. (b) Without accumulator.

Fig. 3: Optimal solution configuration for the extension scenario.

5 Conclusion and Outlook

We presented a more accurate nonlinear model of a hydrostatic transmission system, which we solved using refined solution strategies. Further research could focus on the combinatorial structure of the problem, in order to solve models involving more valves. Furthermore, the investigation of a cost model for the accumulator sizing would be interesting.

Funding This research was funded by Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – project A9 within SFB 805 (Project Number 57157498).

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