No Lorentz Property of M.W. EVANS' $O(3)$–Symmetry Law

— a Remark on a Former Article [1] in this Journal —

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Abstract. The article [1] "On the Nature of the $B^{(3)}$ Field" essentially refers to a hypothesis that was proposed in 1992 by M.W. EVANS: EVANS claimed that a so-called $O(3)$–symmetry of electromagnetic fields should exist due to an additional constant longitudinal "ghost field" $B^{(3)}$ accompanying the well-known transversal plane em waves. EVANS considered this symmetry, a fixed relation between the transversal and the longitudinal amplitudes of the wave, as a *new law of electromagnetics.* In the article [1] in this Journal the authors claim "that the Maxwell-Heaviside theory is incomplete and limited" and should be replaced with EVANS' $O(3)$–theory the center of which is EVANS' $O(3)$–symmetry law. Later on, since 2002, this $O(3)$–symmetry became the center of EVANS' CGUFT which he recently renamed as ECE Theory.

A law of Physics must be invariant under admissible coordinate transforms, namely under Lorentz transforms. A plane wave remains a plane wave also when seen from arbitrary other inertial systems. Therefore, EVANS' $O(3)$–symmetry law should be valid in all inertial systems. To check the validity of EVANS' $O(3)$–symmetry law in other inertial systems, we apply a longitudinal Lorentz transform to EVANS' plane em wave (the ghost field included). As is well-known from SRT and recalled here the transversal amplitude decreases while the additional longitudinal field remains unchanged. Thus, EVANS' $O(3)$–symmetry cannot be invariant under (longitudinal) Lorentz transforms: EVANS' $O(3)$–**symmetry is no valid law of Physics.** Therefore it is impossible to draw any valid conclusions from that *wrong* $O(3)$-hypothesis. Especially the article [1] has no scientific basis.

1. EVANS' $O(3)$–Symmetry

The claim of $O(3)$–symmetry is a central concern of EVANS' considerations since 1992. The reader will find a historical overview in [3; Sect.5] written by A. Lakhtakia. Among a lot of papers EVANS has written five books on "The Enigmatic Photon" that deal with the claimed $O(3)$–symmetry of electromagnetic fields.

In [2; Chap.1.2] EVANS considers a circularly polarized plane electromagnetic wave propagating in z-direction. Using the electromagnetic phase

$$\Phi = \omega t - \kappa z \qquad \text{[2; (1.38)]}$$

where $\kappa = \omega/c$. EVANS describes the wave relative to his complex circular basis [2; (1.41)], see also [4; Appendix 1]. The magnetic field is given by

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$$\mathbf{B}^{(1)} = B^{(0)}\mathbf{q}'^{(1)} = \tfrac{1}{\sqrt{2}}B^{(0)}(\mathbf{i} - i\mathbf{j})e^{i\Phi}\ ,$$

[2;(1.43)]

$$\mathbf{B}^{(2)} = B^{(0)}\mathbf{q}'^{(1)} = \tfrac{1}{\sqrt{2}}B^{(0)}(\mathbf{i} + i\mathbf{j})e^{-i\Phi}\ ,$$

$$\mathbf{B}^{(3)} = B^{(0)}\mathbf{q}'^{(3)} = B^{(0)}\mathbf{k},$$

satisfying Evans' "cyclic $O(3)$–symmetry relations"

$$\mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = iB^{(0)}\mathbf{B}^{(3)*},$$

[2;(1.44)]

$$\mathbf{B}^{(2)} \times \mathbf{B}^{(3)} = iB^{(0)}\mathbf{B}^{(1)*},$$

$$\mathbf{B}^{(3)} \times \mathbf{B}^{(1)} = iB^{(0)}\mathbf{B}^{(2)*}.$$

Due to M.W. Evans' the corresponding electric field is given by

$$\mathbf{E}^{(1)} = -\tfrac{1}{\sqrt{2}}E^{(0)}(i\mathbf{i} + \mathbf{j})\ e^{i\Phi}\ ,$$

[2;(1.85)]

$$\mathbf{E}^{(2)} = \tfrac{1}{\sqrt{2}}E^{(0)}(i\mathbf{i} - \mathbf{j})\ e^{-i\Phi}\ ,$$

$$\mathbf{E}^{(3)} = -iE^{(0)}\mathbf{k}.$$

The relation between $E^{(0)}$ and $B^{(0)}$ is

[2; (1.87)] $$E^{(0)} = cB^{(0)}.$$

We can determine the real representations of the involved fields: Due to $\mathbf{B}^{(2)} = \mathbf{B}^{(1)*}$ and $\mathbf{E}^{(2)} = \mathbf{E}^{(1)*}$ the complex fields [2;(1.43)] and [2;(1.85)] belong to the real fields

$$\mathbf{B} = \mathbf{B}^{(1)} + \mathbf{B}^{(2)} + \mathbf{B}^{(3)} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

and

$$\mathbf{E} = \mathbf{E}^{(1)} + \mathbf{E}^{(2)} + \mathbf{E}^{(3)} = E_x\mathbf{i} + E_y\mathbf{j} + E_z\mathbf{k}.$$

Insertion of [2;(1.43)] and [2;(1.85)] and coefficient matching yields

(1.1) $$B_x = \frac{1}{\sqrt{2}}B^{(0)}\cos\Phi, \qquad B_y = \frac{1}{\sqrt{2}}B^{(0)}\sin\Phi, \qquad B_z = B^{(0)},$$

(1.2) $$E_x = \frac{1}{\sqrt{2}}E^{(0)}\sin\Phi, \qquad E_y = -\frac{1}{\sqrt{2}}E^{(0)}\cos\Phi, \qquad E_z = E^{(0)}.$$

Summing the equations in (1.1) with combination factors $1, \pm i$ and comparing with Eqns.[2;(1.43)] yields

(1.3) $$(B_x + iB_y)(\mathbf{i} - i\mathbf{j}) = 2\mathbf{B}^{(1)}, \qquad (B_x - iB_y)(\mathbf{i} + i\mathbf{j}) = 2\mathbf{B}^{(2)}$$

and therefore for further use in rewriting of the first equation of [2;(1.44)]

(1.4) $$\mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = \frac{1}{2}(B_x + iB_y)(B_x - iB_y)i\mathbf{k} = \frac{1}{2}(B_x^2 + B_y^2)i\mathbf{k}$$

while the last equations of [2;(1.43)] and (1.1) yield

$$B^{(0)}\mathbf{B}^{(3)*} = i\mathbf{k}B^{(0)2} = i\mathbf{k}B_z^2.$$

Thus, one of Evans' "cyclic symmetry relations", the first rule of [2;(1.44)], is equivalent to

(1.5) $$\frac{1}{2}(B_x^2 + B_y^2) = B_z^2.$$

The first two equations of [2;(1.43)] and [2;(1.85)] describe a circularly polarized plane wave propagating in z–direction. The third equations, however, contain EVANS' $O(3)$–**Law** from 1992, saying that the well–known plane wave is always accompanied by a constant longitudinal magnetic "ghost field" $\mathbf{B}^{(3)}$, the *size* of which – *this is important* – is given by the third equation of [2;(1.43)], or by the first equation of [2;(1.44)], which in real formulation is our equation (1.5).

2. The Transformation Behavior of the $O(3)$–Symmetry Law

If EVANS' $O(3)$–Law were a *Law of Physics* then it must be *invariant* under the admissible coordinate transforms, i.e. under Lorentz transforms.

Therefore we consider the wave as observed from other coordinate systems S' in constant motion $\mathbf{v} = v\ \mathbf{k}$ relative to our original Cartesian coordinate system S. The transformation rules for the electromagnetic field are well-known (where $\beta = v/c, \gamma = \sqrt{1-\beta^2}$)):

$$(2.1)\qquad E'_x = \frac{1}{\gamma}(E_x - \beta B_y), \qquad E'_y = \frac{1}{\gamma}(E_y + \beta B_x), \qquad E'_z = E_z,$$

$$(2.2)\qquad B'_x = \frac{1}{\gamma}(B_x + \frac{\beta}{c}E_y), \qquad B'_y = \frac{1}{\gamma}(B_y - \frac{\beta}{c}E_x), \qquad B'_z = B_z.$$

We shall check the first rule of EVANS' $O(3)$–symmetry Law [2;(1.44)] in our equivalent real formulation (1.5). Therefore we are now going to transform the wave (1.1-2) to the coordinate frame S' by means of the transformation rules (2.1-2) to obtain

$$(2.3)\qquad \begin{aligned} B'_x &= \tfrac{1-\beta}{\gamma} B^{(0)}\sqrt{2}\cos\Phi = \tfrac{1-\beta}{\gamma} B_x, \\ B'_y &= \tfrac{1-\beta}{\gamma} B^{(0)}\sqrt{2}\sin\Phi = \tfrac{1-\beta}{\gamma} B_y, \\ B'_z &= B_z\ , \end{aligned}$$

which yields

$$\frac{1}{2}(B'^2_x + B'^2_y) = \frac{1-\beta}{1+\beta}\,\frac{1}{2}(B^2_x + B^2_y) = \frac{1-\beta}{1+\beta}B^2_z = \frac{1-\beta}{1+\beta}B'^2_z \quad < \quad B'^2_z \qquad (0 < \beta < 1).$$

Hence EVANS' first $O(3)$–symmetry relation (1.5) in S', the equation

$$\frac{1}{2}(B'^2_x + B'^2_y) = B'^2_z,$$

is **not** fulfilled:

EVANS' cyclical $O(3)$–symmetry is **not Lorentz invariant** and hence **no law of Physics**.

Therefore, no valid conclusions can be drawn from that wrong $O(3)$-hypothesis.

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