

On the Lorentz Variance of the Claimed $O(3)$ -Symmetry Law

— a Remark on a Former Article [2] in this Journal —

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Abstract. In 1992 M.W. EVANS proposed a so-called $O(3)$ symmetry of electromagnetic fields by adding a constant longitudinal ghost field to the well-known transversal plane em waves. Evans considered this symmetry as a *new law of electromagnetics*. In 2000 he tried to show the Lorentz invariance of $O(3)$ symmetry of electromagnetic fields in this Journal [2]. However, this proof lacks from a simple calculation error. As shall be shown below a correct calculation yields no Lorentz invariance.

This result is of importance as later on, since 2002, this $O(3)$ symmetry became the center of M.W. EVANS' CGUFT which he recently renamed as ECE Theory. – A law of Physics must be invariant under admissible coordinate transforms, namely under Lorentz transforms. Therefore, to check the validity of M.W. EVANS' $O(3)$ symmetry law, we apply a longitudinal Lorentz transform to M.W. EVANS' plane em wave (the ghost field included). As is well-known from SRT and recalled here the transversal amplitude decreases while the additional longitudinal field remains unchanged. Thus, M.W. EVANS' $O(3)$ symmetry cannot be invariant under (longitudinal) Lorentz transforms: **M.W. EVANS' $O(3)$ symmetry is no valid law of Physics.**

A law of Physics must be invariant under admissible coordinate transforms, namely under Lorentz transforms. A plane wave remains a plane wave also when seen from arbitrary other inertial systems. Therefore, EVANS' $O(3)$ symmetry law should be valid in all inertial systems. Therefore to check the validity of EVANS' $O(3)$ symmetry law in other inertial systems, we apply a longitudinal Lorentz transform to EVANS' plane em wave (the ghost field included). As is well-known from SRT and recalled here the transversal amplitude decreases while the additional longitudinal field remains unchanged. Thus, EVANS' $O(3)$ symmetry cannot be invariant under (longitudinal) Lorentz transforms: **EVANS' $O(3)$ symmetry is not a valid law of Physics.**

In the following text quotations from M.W. EVANS' GCUFT book [1] appear with equation labels [1.nn] at the left margin.

1. M.W. EVANS' $O(3)$ hypothesis

The assertion of $O(3)$ symmetry is a central concern of M.W. EVANS' considerations since 1992: He claims that each plane circularly polarized electromagnetic wave \mathbf{B} is accompanied by a constant longitudinal field $\mathbf{B}^{(3)}$, a so-called "ghost field".

Evans considers a circularly polarized plane electromagnetic wave propagating in z-direction, cf. [1; Chap.1.2]. Using the electromagnetic phase

$$[1.38] \quad \Phi = \omega t - \kappa z,$$

where $\kappa = \omega/c$, M.W. EVANS describes the wave relative to his complex circular basis [1;(1.41)]. The magnetic field is given by

$$[1.43/1] \quad \mathbf{B}^{(1)} = \frac{1}{\sqrt{2}} B^{(0)} (\mathbf{i} - i\mathbf{j}) e^{i\Phi},$$

$$[1.43/2] \quad \mathbf{B}^{(2)} = \frac{1}{\sqrt{2}} B^{(0)} (\mathbf{i} + i\mathbf{j}) e^{-i\Phi},$$

$$[1.43/3] \quad \mathbf{B}^{(3)} = B^{(0)} \mathbf{k},$$

satisfying M.W. EVANS' "cyclic $O(3)$ symmetry relations"

$$[1.44/1] \quad \mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = iB^{(0)} \mathbf{B}^{(3)*},$$

$$[1.44/2] \quad \mathbf{B}^{(2)} \times \mathbf{B}^{(3)} = iB^{(0)} \mathbf{B}^{(1)*},$$

$$[1.44/3] \quad \mathbf{B}^{(3)} \times \mathbf{B}^{(1)} = iB^{(0)} \mathbf{B}^{(2)*}.$$

Especially equ.[1.43/3] defines M.W. EVANS' ghost field $\mathbf{B}^{(3)}$, which is coupled by the relations [1.44] with the transversal \mathbf{B} components given by the two eqns.[1.43/1-2].

Evans' **B Cyclic Theorem** is the statement that each plane circularly polarized wave [1.43/1-2] is accompanied by a longitudinal field [1.43/3], and the associated fields fulfil the Cyclic equations [1.44]. Evans considers this $O(3)$ hypothesis as a **Law of Physics**.

2. Checking the Lorentz invariance of the $O(3)$ hypothesis

In the article [2; p.14] M.W. EVANS therefore tries to prove the Lorentz invariance of the $O(3)$ hypothesis [1.44] by referring to the invariance of the vector potential \mathbf{A} under Lorentz transforms. That is a good method obtaining the transform of ED fields if one calculates *correctly*.

So the reader should first check that the vector potentials of the transversal components $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$ of the plane wave under consideration are given by

$$(2.1) \quad \mathbf{A}^{(1)} = \frac{1}{\kappa} \mathbf{B}^{(1)} = \frac{1}{\kappa\sqrt{2}} B^{(0)} (\mathbf{i} - i\mathbf{j}) e^{i\Phi}, \quad \mathbf{A}^{(2)} = \frac{1}{\kappa} \mathbf{B}^{(2)} = \frac{1}{\kappa\sqrt{2}} B^{(0)} (\mathbf{i} + i\mathbf{j}) e^{-i\Phi}$$

while the vector potential of the *longitudinal* field $\mathbf{B}^{(3)}$ is

$$(2.2) \quad \mathbf{A}^{(3)} = \frac{1}{2} \mathbf{B}^{(3)} \times (x\mathbf{i} + y\mathbf{j}).$$

The invariance of the vector potential $\mathbf{A}^{(3)}$ yields the invariance of $\mathbf{B}^{(3)}$ as well for *longitudinal* Lorentz transforms between inertial frames K, K' where K' moves relative to K with velocity $\mathbf{v} \parallel \mathbf{k}$ and $\beta := |\mathbf{v}|/c$.

What M.W. EVANS has obviously ignored in [2]: Frequency ω and wave number κ are *no invariants*. Under *longitudinal* Lorentz transforms we have the well-known Doppler effect:

$$(2.3) \quad \omega' = \sqrt{\frac{1-\beta}{1+\beta}} \omega, \quad \kappa' = \sqrt{\frac{1-\beta}{1+\beta}} \kappa.$$

Therefore the invariance of the vector potentials doesn't transfer to the transversal field components $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$.

Due to the invariance of \mathbf{A} we obtain from (2.1)

$$(2.4) \quad \mathbf{B}'^{(1)} \times \mathbf{B}'^{(2)} = \kappa'^2 \mathbf{A}'^{(1)} \times \mathbf{A}'^{(2)} = \frac{1-\beta}{1+\beta} \kappa^2 \mathbf{A}^{(1)} \times \mathbf{A}^{(2)} = \frac{1-\beta}{1+\beta} \mathbf{B}^{(1)} \times \mathbf{B}^{(2)},$$

i.e. the expression $\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}$ at the left hand side of [1.44/1] does not remain invariant while $\mathbf{B}^{(3)}$ is invariant due to (2.2). Hence equ.[1.44/1], if valid in the inertial system K , *cannot be valid* also in the inertial system K' : Evans' cyclical symmetry [1.44] is *not Lorentz invariant and so cannot be a Law of Physics*.

References

- [1] M.W. EVANS, Generally Covariant Unified Field Theory, the geometrization of physics; Arima 2006
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- [3] G.W. BRUHN AND A. LAKHTAKIA, Commentary on Myron W. Evans' paper "The Electromagnetic Sector ...",
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