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# BOOK OF ABSTRACTS

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### Diffuse Interface Models for Two-Phase Flows with Different Densities

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**Abstract.** We discuss different models for a two-phase flow of two immiscible, incompressible fluids in the case when the densities of the fluids are different. In particular we will present a new thermodynamically consistent diffuse interface model and compare it with the known models. Such models were introduced to describe the flow when singularities in the interface, which separates the fluids, (droplet formation/coalescence) occur. The fluids are assumed to be macroscopically immiscible, but a partial mixing in a small interfacial region is assumed. We will present recent results on the mathematical analysis of these models.

### Parabolic transmission boundary value problems

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**Abstract.** We discuss linear parabolic equations of reaction diffusion type in a multiphase setting. The phase boundaries, on which transmission conditions are imposed, are allowed to touch each other as well as the boundary of the domain. The latter may have edges and corners, and Dirichlet and Neumann conditions may be imposed on adjacent components. In addition, the boundary and the phase boundaries can move. This leads to non cylindrical space - time domains. In this general setting we provide an optimal maximal regularity result in suitably weighted Sobolev spaces. It is the basis for the study of related nonlinear problems.

### Representations for solutions of second-order elliptic equations via generalized Cauchy-Riemann systems

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**Abstract.** I will give an account of a series of works with Andreas Axelsson. I will explain the method to represent solutions towards solvability of Dirichlet problem. The main unknown function is the conormal gradient of the solution which solves a generalized Cauchy-Riemann system. This system is handled by a functional and operational calculus approach based on the full solution of the Kato problem and its extensions when the coefficients are non smooth.

### Integral resolvent conditions

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**Abstract.** A typical integral resolvent condition for an operator $A$ on a Banach space $X$ involves finiteness and estimates of integrals involving powers of the resolvent of $A$ along some family of parallel lines. The talk will discuss results linking these conditions to topics such as generation and stability of semigroups, functional calculus and perturbations.

### Inverse Problems of Calderón Type

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**Abstract.** We discuss certain generalizations of the classical Calderón problem, which has its origins in electrical impedance tomography: Can the conductivity of an inhomogeneous, isotropic body be uniquely recovered from measurements of the normal component of the current flux corresponding to a voltage potential on only the surface of this body or parts of its surface? In other words, the question is, whether the variable coefficient of the second order elliptic differential expression $\nabla \cdot \gamma \nabla$ on a bounded domain $\Omega \subset \mathbb{R}^n$ can be recovered by knowledge of an associated Dirichlet-to-Neumann map at the boundary $\partial\Omega$ of the domain or on parts of $\partial\Omega$. This question has been answered positively when, for instance, boundary measurements on the whole surface are given under different a priori assumptions on $\gamma$. The main objective of this lecture is to consider more general elliptic differential expressions of the form $\mathcal{L} = \sum \partial_j a_{jk} \partial_k + a$ with variable coefficients $a_{jk}$ and $a$ and to assume Dirichlet and Neumann data to be measured only on an arbitrarily small nonempty open subset $\omega$ of the surface. Although uniqueness of the coefficients can not be expected, it will be shown that milder forms of uniqueness results can still be obtained: The Dirichlet operator associated to $\mathcal{L}$ is uniquely determined (up to unitary equivalence) and its complete spectral information can be reconstructed from the Dirichlet-to-Neumann map on $\omega$. The lecture is based on joint work with Jonathan Rohleder.

### Weak solutions to stochastic geometric wave equations

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**Abstract.** Stochastic wave equations with values in Riemannian manifolds, also called stochastic geometric wave equations (SGWE) were introduced by the speaker and M Ondrejat in 2007. They proved the existence and uniqueness of global strong regular solutions defined on the one-dimensional Minkowski space $\mathbb{R}^{1+1}$ in the case when the target manifold $M$ is an arbitrary compact Riemannian manifold. In this talk I will describe results obtained in joint works with M Ondrejat about the existence and uniqueness of global weak solutions defined on a Minkowski space $\mathbb{R}^{1+d}$ when either (i) the target manifold $M$ is a compact Riemannian homogeneous space or (ii) $d = 1$.

### Regularity for solutions of the total variation denoising problem

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### Maximal regularity and the Local Inverse Function Theorem

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## Functional calculus in an annulus

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**Abstract.** Let us consider the annulus $\mathcal{A}_R = \{z \in \mathbb{C}\,; |z| \le R \text{ and } |z^{-1}| \le R\}$, $R > 1$ and a bounded operator $A$ on a Hilbert space $H$ with spectrum contained in $\mathcal{A}_R$. The annulus is said to be a $K$-spectral set for $A$, if the inequality

$$\|r(A)\| \le K \sup\{|r(z)|\,; z \in \mathcal{A}_R\}$$

holds for all bounded rational functions on $\mathcal{A}_R$. We discuss some estimates of the constant $K$ under different hypotheses, for instance "$\|A\| \le R \quad \text{and} \quad \|A^{-1}\| \le R$", or "$w(A) \le R \quad \text{and} \quad w(A^{-1}) \le R$". Here $w(A) := \sup\{\langle Av, v\rangle\,; \|v\| = 1, v \in H\}$ denotes the numerical radius of the operator $A$.

References

[1] C. Badea, B. Beckermann and M. Crouzeix, Intersections of several disks of the Riemann sphere as $K$-spectral sets, Com. Pure Appl. Anal. **8, 1**, 37-54 (2009).

## The Faber-Krahn inequality for Robin problems

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**Abstract.** The classical Faber-Krahn inequality states that the first eigenvalue of the Laplacian with Dirichlet boundary conditions is minimal for the ball if we look at all domains in $\mathbb{R}^N$ of the same volume. This is a classical result from mathematical physics going back to a conjecture by Rayleigh 1877. The usual proof of such isoperimetric inequalities is by symmetrisation methods. We discuss a similar result for the $p$-Laplacian with Robin (third) boundary conditions. In that case symmetrisation methods do not seem to work since the first eigenfunction is not in general constant on the boundary of the domain. We explain what the problem is and sketch an alternative proof of the long standing conjecture going back probably to Krahn. The ideas originate from [1], where a partial proof was given for $N = 2$ using ideas from complex analysis. The results are partially joint work with James Kennedy and Dorin Bucur (see [2, 4, 3]).

References

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[2] D. Bucur and D. Daners, *An alternative approach to the Faber-Krahn inequality for Robin problems*, Calc. Var. Partial Differential Equations **37** (2010), 75–86.

[3] D. Daners, *A Faber-Krahn inequality for Robin problems in any space dimension*, Math. Ann. **335** (2006), 767–785.

[4] D. Daners and J. Kennedy, *Uniqueness in the Faber-Krahn inequality for Robin problems*, SIAM J. Math. Anal. **39** (2007), 1191–1207.

### Elliptic equations in open subsets of infinite dimensional Hilbert spaces

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**Abstract.** We consider the equation

$$\lambda\varphi - L\varphi = f \tag{1}$$

where $\lambda \geq 0$ and $L$ is the Kolmogorov operator

$$L\varphi = \frac{1}{2}\mathrm{Tr}[CD^2\varphi] + \langle Ax + b(x), D\varphi\rangle$$

defined in an open subset $\mathscr{O}$ of a Hilbert space $H$, equipped with suitable boundary conditions on $\partial\mathscr{O}$. $A$ is the infinitesimal generator of a $C_0$ semigroup in $H$, $C$ is a bounded positive operator in $H$ and $b$ is a nonlinear perturbation.
We discuss some existence and regularity results of the solution $u$ of (1) when $f \in L^2(\mathscr{O}, \mu)$ and $\mu$ is the invariant measure of $L$.

### Maximal $L^p$-regularity for parabolic mixed-order systems

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**Abstract.** We study maximal $L_p$-regularity for a class of pseudodifferential mixed order systems on a space-time cylinder $\mathbb{R}^n \times [0,\infty)$ or $X \times [0,\infty)$, where $X$ is a closed smooth manifold. To this end we construct a calculus of Volterra pseudodifferential operators and characterize the parabolicity of a system by the invertibility of certain associated symbols. A parabolic system is shown to induce isomorphisms between suitable $L_p$-Sobolev spaces of Bessel potential or Besov type. In the case $X \times [0,\infty)$, the inverse of a parabolic system belongs to the calculus again. Applications include, e.g., time-dependent Douglis-Nirenberg systems and linear systems arising in the study of boundary value problems with dynamic boundary conditions.

This talk is based on joint work with J. Seiler (Loughborough).

References

[1] R. Denk and J. Seiler, On the maximal $L_p$-regularity of parabolic mixed order systems, Preprint Konstanzer Schriften in Mathematik **266** (2010).

### On a Paley-Littlewood inequality

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**Abstract.** In [1] a Littlewood-Paley inequality and its application to an $L^p$-estimate of the gradient of the heat kernel are proved. These estimates are crucial tools in the development of a theory of parabolic stochastic partial differential equations in [2]. We generalize these inequalities so that they can be applied to stochastic integrodifferential equations. (Joint work with S. O. Londen.)

References

[1] N. V. Krylov, A parabolic Littlewood-Paley inequality with applications to parabolic equations, Topological Methods in Nonlinear Analysis, Journal of the Juliusz Schauder Center 4 (1994), 355–364.

[2] N. V. Krylov, An analytic approach to SPDEs. In *Stochastic Partial Differential Equations: Six Perspectives*, R.A. Carmona and B. Rozovskii, eds., A.M.S. Mathematical Surveys and Monographs, 64 (1999), 185–242.

### Douglis-Nirenberg Systems and Analytic Semigroups

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**Abstract.** We study mixed order parameter-elliptic boundary value problems with boundary conditions of a certain structure. For such operators, we prove the analyticity of the semigroup via resolvent estimates in $L^p$-based Sobolev spaces of suitable order; and we present an application of this theory to studies of the particle transport in a semi-conductor.

### Doubly nonlinear evolution equations of second order: Existence and fully discrete approximation

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**Abstract.** The initial-value problem for doubly nonlinear evolution equations of the type $u'' + Au' + Bu = f$ is studied. The time-dependent operator $A$ is assumed to satisfy a certain growth condition and is supposed to be the sum of a monotone, coercive and hemicontinuous principal part and a strongly continuous perturbation. The operator $B$ is supposed to be the sum of an operator that induces an inner product and a non-monotone perturbation fulfilling a certain local Hölder-type continuity condition. Examples are viscous regularisations of equations arising in elasticity. For a full discretisation combining a modification of the Störmer-Verlet method with an inner approximation scheme, weak convergence of piecewise polynomial prolongations of the discrete solutions are proven. These results also imply the existence of a weak solution. For the time discretisation, also variable time grids are allowed as long as the deviation from equidistance is not too large.
A crucial step in the convergence proof is the equivalence with an integro-differential equation of parabolic type. In the case of non-monotone perturbations, a priori estimates in fractional Sobolev-Slobodetskii spaces are a prerequisite for the necessary compactness argument.

This is joint work with Mechthild Thalhammer (Innsbruck, Austria).

### On some Geometric Evolution Equations in Hydrodynamics

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**Abstract.** A geometric interpretation of the periodic Degasperis-Procesi equation as the geodesic flow of a right invariant symmetric linear connection on the diffeomorphism group of the circle is presented. Furthermore it is shown that there is no inertia operator on the tangent bundle inducing a Riemannian connection on the diffeomorphism group with the above-mentioned property.

It is also disclosed that for any evolution within the family of $b$-equations there is neither gain nor loss of the spatial regularity of solutions. This in turn is crucial in order to realize the Degasperis-Procesi and the Camassa-Holm equation as an ODE on the Fréchet space of all smooth functions on the circle.

References


[1] V. I. Arnold, Sur la géométrie différentielle des groupes de Lie de dimension infinie et ses applications à l'hydrodynamique des fluides parfaits, *Ann. Inst. Fourier*, **16**, 319–361 (1966).

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### Random data Cauchy theory for the incompressible three dimensional Navier–Stokes equations


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**Abstract.** This is a joint work with Ting Zhang. We study the existence and uniqueness of strong solutions for the incompressible Navier–Stokes equations with $L^2$ initial data and periodic space domain $\mathbb{T}^3$. After a suitable randomization, we are able to construct local strong solution for a large set of initial data in $L^2$. Furthermore, if $\|u_0\|_{L^2}$ is small, we show that the probability for the global existence and uniqueness of the solution is large.


### Operator splitting for non-autonomous equations


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**Abstract.** We consider abstract non-autonomous equations of the form

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t}u(t) = (A(t) + B(t))u(t), \quad t \geq s \in \mathbb{R}, \\ u(s) = x. \end{cases}$$

Provided the initial value problems associated to $A(\cdot)$ and $B(\cdot)$ are well-posed on the Banach space $X$, one wishes to recover the solution of the original problem from these "sub-solutions". In numerics this procedure is often called operator splitting. Under stability assumption of Kato type we prove convergence of such schemes, more precisely analogues of Lie-Trotter product formulas, for the evolution families generated by $A(\cdot)$ and $B(\cdot)$. The abstract results are then applied to heat equations with non-autonomous potential, and to non-autonomous delay equations. In these cases we even obtain convergence rates. The talk is based on a joint work with A. Bátkai, P. Csomós, G. Nickel.

### On suitable weak solutions to the compressible Navier-Stokes system

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**Abstract.** We introduce a new concept of suitable weak solution to the Navier-Stokes system describing the motion of a compressible viscous fluid. Several properties of these solutions are discussed: weak-strong uniqueness issues, conditional regularity, stability of approximate schemes.

References

[1] E.Feireisl, A.Novotný, Y. Sun, Suitable weak solutions to the Navier-Stokes equations of compressible viscous fluids, preprint.

### Weak Neumann implies Stokes

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**Abstract.** Let $\Omega \subset \mathbb{R}^n$ be a domain with uniform $C^3$-boundary. We show that the Stokes operator generates an analytic semigroup on $L^p(\Omega)$ provided that the weak Neumann problem is solvable in $L^p(\Omega)$. In this case, the Stokes operator also satisfies maximal regularity estimates.

### Planar motion by anisotropic curvature derived from singular interfacial energy

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**Abstract.** A curvature flow is important to describe motion of phase boundaries in materials science. Because of anisotropy of crystal structure it often happens that the interfacial energy is anisotropic and sometimes anisotropy is too strong so that the interfacial energy density is not smooth and allows jumps of the derivative. A typical example is a crystalline curvature flow.
Motion by singular interfacial energy is regarded as a singular diffusion equation (if the interfacial energy is convex.) Another typical example of singular diffusion equations is the total variation flow.
Unlike the total variation flow, motion of curves by singular interfacial energy is not necessarily of divergence type. For such equations it is often nontrivial to construct a solution for general initial data.
In this talk we discuss some recent topics related to singular diffusion equations including crystalline curvature flows.

### Blow-up arguments and the Navier-Stokes equations

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**Abstract.** A blow-up argument is a very strong and flexible tool to prove regularity of solutions of nonlinear partial differential equations which goes back to the study of the minimal surfaces by E. De Giorgi in 1961. It is also useful to obtain a priori upper bounds for solutions.

In this talk we give two new applications of this method for the Navier-Stokes and Stokes equations. As the first application we give a geometric non-blow-up criterion on the direction of the vorticity for the three dimensional Navier-Stokes flow whose initial data is just bounded and may have infinite energy. We prove that under a restriction on behavior in time (type I condition) the solution does not blow up if the vorticity direction is uniformly continuous at place where vorticity is large. This improves the Lipschitz regularity condition for the vorticity direction first introduced by P. Constantin and C. Fefferman in 1993 for finite energy weak solutions.
In a second application we are able to prove that the Stokes operator (with the Dirichlet boundary condition) generates an analytic semigroup in the space of bounded, uniformly continuous functions at least when a domain occupied with fluid is a bounded domain in the Euclidean space. This is a long standing open problem and it was only known when the domain is a half space where an explicit solution formula is available. A conventional method to reduce the problem to a half space does not work since various elliptic and parabolic estimates are not valid for sup-norms so perturbed terms cannot be controlled. Also, it seems difficult to extend the method available for the Laplace operator. In our method we reduce the problem to a half space problem by a blow-up argument to get a priori estimates.
The first part is a joint work with H. Miura (Osaka University) while the second part is a joint work with my student Ken Abe (University of Tokyo). The second part is a work in progress while the preprint of the first part is available at *http://eprints3.math.sci.hokudai.ac.jp/2045/*.

**Instantaneous blowup and related nonexistence issues**

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**Abstract.** Parabolic PDEs describe various physical phenomena. Positive solutions are of special interest as the solution might represent a temperature, density or concentration. But if an ingredient of the PDE is "too singular", positive solutions might not exist at all. We shall discuss new results as well as the history of this topic, which initially involved linear problems on (a domain in) $\mathbb{R}^n$. The theory has been extended to nonlinear equations, problems in which $\mathbb{R}^n$ is replaced by various nonabelian groups (e.g., Carnot groups), and problems involving the Ornstein-Uhlenbeck and Kolmogorov equations.

**The Use of Nonlinear Diffusions in Image Processing**

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**Abstract.** Ever since the introduction of the Mumford-Shah and the Perona-Malik models, variational and PDE-based methods have enjoyed a remarkable success as low level tools for performing various tasks of Image Processing. Often such models exhibit very interesting and non-trivial mathematical features. In this talk we shall give an overview on the use of such methods with particular focus on nonlinear diffusions of second and fourth order. Results of mathematical nature will be presented along with numerical experiments which motivated and/or confirmed theoretical predictions and insights.

### Some remarks on a decade of functional calculus

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**Abstract.** In this talk I want to review some of the developments of functional calculus theory in the last decade, with a focus on my own work and the inspiration it took from the close contact with Wolfgang Arendt.

### The square root of divergence form operators with mixed boundary conditions

Robert Haller-Dintelmann (haller@mathematik.tu-darmstadt.de)

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**Abstract.** We show that the square roots of divergence form operators of second order are isomorphisms between the appropriate $W^{1,p}$-space and $L^p(\Omega)$ for all $p \geq 2$, even if

- the boundary conditions are mixed Dirichlet/Neumann or Dirichlet/Robin
- the domain $\Omega$ is just a bounded Lipschitz domain and
- the coefficients are real-valued, elliptic and just bounded.

Some applications, e.g. to parabolic problems, are also presented.

This is joint work with Joachim Rehberg (WIAS, Berlin).

### Stationary solutions of the Navier-Stokes equations in the exterior of a rotating obstacle

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**Abstract.** We consider the motion of a viscous incompressible fluid around an obstacle that is rotating with constant speed and moving with constant speed and direction. The underlying equations describing the fluid flow are given by the Navier-Stokes equations in a time depending domain. After a change of coordinates it is possible to rewrite the equations in a time independent domain. In this talk we study existence and uniqueness for stationary weak solutions of the transformed equations in Lorentz spaces.

The presented results are based on a joint work with Hyunseok Kim and Hideo Kozono.

### 2D flow around a rotating obstacle

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**Abstract.** The exterior steady problem in 2D is quite difficult. I would like to report that the situation becomes less difficult when the obstacle is rotating. This is because the rotation in 2D implies better asymptotic behavior of the flow at infinity. If the obstacle is a rotating disk, it is easy to see that there is a Navier-Stokes flow, as an exact solution, which decays like $1/|x|$.

In this talk I consider the plane Stokes flow around a rotating obstacle without any symmetry and show that it decays like $1/|x|$ provided the support of the external force is bounded. What is remarkable is that a rotating profile is found in the leading term of the flow (while it is not in the leading term but in the second term for 3D case [1]).

References

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### Return to the treasure island of Bourgain

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**Abstract.** Without question, the paper [1] of Bourgain was the principal source of inspiration for the boom of developments around operator-valued Fourier multipliers, $R$-boundedness, and maximal regularity around the turn of the millennium. Among the mere lemmas of that paper we find such treasures as the introduction of $R$-boundedness, the vector-valued Stein inequality, and finally, the notorious Lemma 10. This lemma was used by Bourgain in order to get his paper's sharpest results for vector-valued singular integrals, but it was not yet needed for the vector-valued Marcinkiewicz multiplier theorem, nor the major part of the later operator-valued developments. However, the possibility to obtain sharper results for multipliers with Lemma 10 was realised by Girardi and Weis [2], and subsequently it was used as a decisive tool in the operator-valued $T1$ theorem of Weis and myself [3], as well as other related problems of vector-valued harmonic analysis. But unlike the rest of Bourgain's lemmas, which became exposed and more approachable over the years, Lemma 10 remained an unopened magic box, only known from outside for its immense power.
The aim of this talk is to reveal the working engine of this fundamental device of vector-valued Fourier analysis. This will be done with the help of random dyadic cubes introduced by Nazarov, Treil and Volberg [4], which allow us to circumvent the most difficult combinatorial core of Bourgain's original proof.

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### Fourier multipliers, square functions, and the wavelet transform

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### Parabolic equations with rough initial data

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**Abstract.** We study parabolic equations with rough initial data. Typical examples include surface diffusion with Lipschitz initial data, the harmonic map heat flow with $BMO$ initial data and the Ricci flow with measurable metrics as initial data. Typically we consider a small perturbation of smooth data in the space of Lipschitz functions, $L^\infty$ or $BMO$ as initial data.

References

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### On the chain rule functional equation for differential functions

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### Global compensated compactness theorem for general differential operators of first order

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**Abstract.** Let $A_1(x,D)$ and $A_2(x,D)$ be differential operators of the first order acting on $l$-vector functions $u=(u_1,\cdots,u_l)$ in a bounded domain $\Omega\subset\mathbb{R}^n$ with the smooth boundary $\partial\Omega$. We assume that the $H^1$-norm $\|u\|_{H^1(\Omega)}$ is equivalent to $\sum_{i=1}^2\|A_iu\|_{L^2(\Omega)}+\|B_1u\|_{H^{\frac12}(\partial\Omega)}$ and $\sum_{i=1}^2\|A_iu\|_{L^2(\Omega)}+\|B_2u\|_{H^{\frac12}(\partial\Omega)}$, where $B_i=B_i(x,\nu)$ is the trace operator onto $\partial\Omega$ associated with $A_i(x,D)$ for $i=1,2$ which is determined by the Stokes integral formula ($\nu$: unit outer normal to $\partial\Omega$). Furthermore, we impose on $A_1$ and $A_2$ a cancellation property such as $A_1A_2'=0$ and $A_2A_1'=0$, where $A_i'$ is the formal adjoint differential operator of $A_i(i=1,2)$. Suppose that $\{u_m\}_{m=1}^\infty$ and $\{v_m\}_{m=1}^\infty$ converge to $u$ and $v$ weakly in $L^2(\Omega)$, respectively. Assume also that $\{A_1u_m\}_{m=1}^\infty$ and $\{A_2v_m\}_{m=1}^\infty$ are bounded in $L^2(\Omega)$. If either $\{B_1u_m\}_{m=1}^\infty$ or $\{B_2v_m\}_{m=1}^\infty$ is bounded in $H^{\frac12}(\partial\Omega)$, then it holds that $\int_\Omega u_m\cdot v_m dx\to\int_\Omega u\cdot v dx$. We also discuss a corresponding result on compact Riemannian manifolds with boundary.

### A few results on VMO spaces and linear and fully nonlinear elliptic and parabolic equations

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**Abstract.** We will discuss solvability issues in Sobolev spaces of second-order linear equations with VMO or even less regular leading coefficients in divergence and nondivergence form with possibly growing lower order coefficients. The main emphasis will be on equations in the whole space. A theorem on Bellman's equations with VMO "coefficients" will be also discussed.

**On optimal $L^p$-$L^q$ estimates for parabolic boundary value problems.**

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**Abstract.** We study boundary value problems that are elliptic with a parameter in an $L^q(\Omega)$-setting. We present a modification of Davies' method for these problems and show that, if the coefficients of the domain operator are only bounded and measurable, then the natural norm bound on the full inhomogeneous resolvent problem self-improves to exponential off-diagonal estimates for the solution operators. These off-diagonal estimates are used to enlarge the $L^q$-scale and to establish optimal $L^p$-$L^q$-estimates for the corresponding parabolic problem, including inhomogeneous boundary data. As an application we derive new results for operators with VMO-coefficients.

**Global attractors for 2-D Kirchoff-Boussinesque evolutions**

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**Abstract.** Dynamics for a class of nonlinear 2D Kirchhoff-Boussinesq models will be considered. These nonlinear plate models are characterized by a presence of a nonlinear source that alone leads to finite-time blow up of solutions. In order to counteract, restorative forces are introduced, which however are of a supercritical nature. This raises natural problems related to existence and wellposedness of weak solutions.

It is shown that finite energy solutions do exist globally and the corresponding flow generates a nonlinear semigroup. The proof is based on logarithmic control of the lack of Sobolev's embedding along with appropriate "negative norm estimates" [3].

In addition to wellposedness, long time behavior is analyzed. It is shown that there exists global attractor corresponding to both weak and strong solutions. The dimension of the said attractor is finite and it displays $C^\infty$ smoothness. Thus, the long time dynamics of a supercritically forced hyperbolic -like flow is shown to enjoy finite dimensional and smooth structures.

These results are obtained by exploiting recently introduced abstract theory for atractiveness [1] along with the method introduced in [2] .

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[3] I. Chueshov and I. Lasiecka, *On Global attractor for 2D Kirchoff-Boussinesq model with supercritical nonlinearity* , Communications on PDE's, to appear 2010.

**The index formula and the spectral shift function for relatively trace class perturbations**

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**Abstract.** We compute the Fredholm index, $\operatorname{index}(\boldsymbol{D_A})$, of the operator $\boldsymbol{D_A} = (d/dt)+\boldsymbol{A}$ on $L^2(\mathbb{R};\mathcal{H})$ associated with the operator path $\{A(t)\}_{t=-\infty}^{\infty}$, where $(\boldsymbol{A}f)(t) = A(t)f(t)$ for a.e. $t \in \mathbb{R}$, and appropriate $f \in L^2(\mathbb{R};\mathcal{H})$, via the spectral shift function $\xi(\,\cdot\,; A_+, A_-)$ associated with the pair $(A_+, A_-)$ of asymptotic operators $A_\pm = A(\pm\infty)$ on the separable complex Hilbert space $\mathcal{H}$ in the case when $A(t)$ is generally an unbounded (relatively trace class) perturbation of the unbounded self-adjoint operator $A_-$.

We derive a formula (an extension of a formula due to Pushnitski [1]) relating the spectral shift function $\xi(\,\cdot\,; A_+, A_-)$ for the pair $(A_+, A_-)$, and the corresponding spectral shift function $\xi(\,\cdot\,; \boldsymbol{H}_2, \boldsymbol{H}_1)$ for the pair of operators $(\boldsymbol{H}_2, \boldsymbol{H}_1) = (\boldsymbol{D_A D_A^*}, \boldsymbol{D_A^* D_A})$ in this relative trace class context,

$$\xi(\lambda; \boldsymbol{H}_2, \boldsymbol{H}_1) = \frac{1}{\pi} \int_{-\lambda^{1/2}}^{\lambda^{1/2}} \frac{\xi(\nu; A_+, A_-)\, d\nu}{(\lambda - \nu^2)^{1/2}} \text{ for a.e. } \lambda > 0.$$

This formula is then used to identify the Fredholm index of $\boldsymbol{D_A}$ with $\xi(0; A_+, A_-)$. In addition, we prove that $\operatorname{index}(\boldsymbol{D_A})$ coincides with the spectral flow $\operatorname{SpFlow}(\{A(t)\}_{t=-\infty}^{\infty})$ of the family $\{A(t)\}_{t\in\mathbb{R}}$ and also relate it to the (Fredholm) perturbation determinant for the pair $(A_+, A_-)$:

$$\operatorname{index}(\boldsymbol{D_A}) = \operatorname{SpFlow}(\{A(t)\}_{t=-\infty}^{\infty}) = \xi(0; A_+, A_-) = \pi^{-1} \ln(\det_{\mathcal{H}}(A_+ A_-^{-1})).$$

We also provide some applications in the context of supersymmetric quantum mechanics to zeta function and heat kernel regularized spectral asymmetries and the eta-invariant.

This is a joint work with F. Geztesy, A. K. Makarov, F. Sukochev, and Y. Tomilov.

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**On some stochastic integral equations**

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**Asymptotic behaviour in time periodic parabolic problems with unbounded coefficients**

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**Abstract.** In this talk we consider a class of nonautonomous elliptic operators $\mathscr{A}$ with unbounded and time periodic coefficients defined in $\mathbb{R}\times\mathbb{R}^N$. Under rather general assumptions on the coefficients, the Cauchy problem $u(s,\cdot)=f\in C_b(\mathbb{R}^N)$ for the parabolic equation $D_tu=\mathscr{A}u$ admits a unique bounded classical solution $u$. This allows to associate an evolution family $\{G(t,s)\}$ with $\mathscr{A}$, in a natural way. Under somewhat stronger assumptions on the coefficients of the operator $\mathscr{A}$, we show that there exists a unique $T$-periodic evolution system of measures associated with the operator $G(t,s)$, i.e., there exists a unique family $\{\mu_t : t\in\mathbb{R}\}$ of probability measures such that $\mu_{t+T}=\mu_t$ for any $t\in\mathbb{R}$ and

$$\int_{\mathbb{R}^N} G(t,s)f d\mu_t=\int_{\mathbb{R}^N} f d\mu_s,\qquad s<t,\ \ f\in C_b(\mathbb{R}^N).$$

This latter property allows to extend the operator $G(t,s)$ to a contraction from $L^p(\mathbb{R}^N,\mu_s)$ to $L^p(\mathbb{R}^N,\mu_t)$ for any such $s$ and $t$ and any $p\in[1,+\infty)$.
We present some results on the asymptotic behavior of the evolution operator $G(t,s)$, as $|t-s|\to+\infty$, in the previous $L^p$-spaces. Our results generalize and improve asymptotic behavior results for Markov semigroups having an invariant measure. We also study spectral properties of the realization of the parabolic operator $u\mapsto\mathscr{A}(t)u-u_t$ in suitable $L^p$-spaces related to a probability measure constructed starting from the $T$-periodic evolution system of measures.

This talk is based on recent results obtained in collaboration with A. Lunardi and A. Zamboni (Parma).

**Dirichlet problems for Ornstein-Uhlenbeck operators in Hilbert spaces**

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**Abstract.** We consider a family of self-adjoint Ornstein–Uhlenbeck operators $\mathcal{L}_\alpha$ in an infinite dimensional Hilbert space $H$ having the same gaussian invariant measure $\mu=\mathcal{N}_Q$,

$$\mathcal{L}_\alpha\varphi(x)=\frac{1}{2}\,\mathrm{Tr}\,[Q^{1-\alpha}D^2\varphi(x)]-\frac{1}{2}\langle x,Q^{-\alpha}D\varphi(x)\rangle,$$

where $Q\in\mathcal{L}(H)$ is a symmetric positive operator with finite trace, and $0\le\alpha\le1$.
We study the Dirichlet problem for the equation $\lambda\varphi-\mathcal{L}_\alpha\varphi=f$ in a closed set $K\subset H$, with $f\in L^2(K,\mu)$. Its variational solution, easily provided by the Lax—Milgram theorem, can be represented by means of the transition semigroup stopped to $K$, as in finite dimensions.
We address two problems: the meaning of the Dirichlet boundary condition and the regularity of the solution $\varphi$ (which belongs to a Sobolev space $W^{1,2}_\alpha(K,\mu)$ by definition) of the Dirichlet problem.
Concerning the boundary condition we consider both irregular and regular boundaries. In the first case we content to have a solution whose null extension outside $K$ belongs to $W^{1,2}_\alpha(H,\mu)$. In the second case we exploit the Malliavin's theory of surface integrals, to give a meaning to the trace of $\varphi$ at $\partial K$ and to show that it vanishes, as it is natural.

Concerning regularity, we can prove interior $W^{2,2}_{\alpha}$ regularity results. Regularity up to the boundary is much more complicated; however we have some partial results. For instance, we can treat the case $\alpha = 0$ for the unit ball and for halfspaces $K = \{x \in H : \langle b, x\rangle = 1\}$ with $b \in H$, $\|b\| = 1$.
The talk is based on joint works with G. Da Prato ([1]).

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## Regularity results on the relativistic heat equation

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**Abstract.** To correct the infinite speed of propagation of the classical diffusion equation Ph. Rosenau proposed the tempered diffusion equation

$$u_t = \nu \operatorname{div} \left( \frac{uDu}{\sqrt{u^2 + \frac{\nu^2}{c^2}|Du|^2}} \right). \tag{1}$$

Equation (1) was derived by Y. Brenier by means of Monge-Kantorovich's mass transport theory and he named it as the *relativistic heat equation*. In [1], we have proved existence and uniqueness of entropy solutions for the Cauchy problem for the quasi-linear parabolic equation $u_t = \operatorname{div} \mathbf{a}(u, Du)$, where $\mathbf{a}(z, \xi) = \nabla_\xi f(z, \xi)$ and $f$ being a function with linear growth as $\|\xi\| \to \infty$, satisfying other additional assumptions. In particular, this class includes the relativistic heat equation (1) and some flux limited diffusion equations used in the theory of radiation hydrodynamics. We also have studied the evolution of the support of entropy solutions of relativistic heat equation. In [2] we have established the following result. *"Let $\Omega$ be an open bounded set in $\mathbb{R}^N$. Let $u_0 \in (L^1(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N))^+$ with support equal to $\overline{\Omega}$. Assume that given any closed set $F \subseteq \Omega$, there is a constant $\alpha_F > 0$ such that $u_0 \geq \alpha_F$ in $F$. Then, if $u(t)$ is the entropy solution of the Cauchy problem for the equation (1) with $u_0$ as initial datum, we have that*

$$\operatorname{supp}(u(t)) = \overline{\Omega} \oplus \overline{B_{ct}(0)} \quad \text{for all } t \geq 0."$$

In this lecture we present some results ([3]) on partial regularity for the entropy solution $u(t)$ of the Cauchy problem (1). Under some assumptions on the initial condition $u_0$, we prove that $u_t(t)$ is a Radon measure in $\mathbb{R}^N$. Moreover, if $u_0$ is log-concave inside its support $\Omega$, $\Omega$ being a convex set, then we show the solution $u(t)$ is also log-concave in its support $\Omega(t) = \Omega + B_{ct}(0)$. This implies its smoothness in $\Omega(t)$. In that case we can give a simpler characterization of the notion of entropy solution.
We also present some recent results ([4]) on the existence of local in time smooth (inside its support) radial solutions of the relativistic heat equation obtained by means of the theory developed by Da Prato-Grisvard and Angenent.

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## Potential maps for de Rham complexes on Lipschitz domains

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**Abstract.** On a domain which is starlike with respect to a ball, integral operators related to the classical Poincaré path integral serve as potential operators for de Rham complexes without boundary conditions, and the dual class generalizing Bogovskiĭ-type operators work for de Rham complexes with full Dirichlet boundary conditions. Such operators were intoduced by Mitrea, and further studied by Mitrea, Mitrea and Monniaux. In joint work with Costabel, we prove that these operators are pseudodifferential operators of order $-1$, and thus obtain further regularity results for these de Rham complexes, for example in Hardy spaces. For bounded Lipschitz domains, the same regularity results hold, and, moreover, the cohomology spaces can always be represented by smooth forms.
In recent work with Costabel and Taggart, we show that kernels constructed by Chang, Krantz and Stein can be used to form potential maps $T$ for de Rham complexes on unbounded special Lipschitz domains $\Omega$, and thus obtain similar regularity results. We also show how the operator $T$ can be used to provide atomic decompositions of Hardy spaces $H^p$ of exact forms with support in $\overline{\Omega}$ whenever $n/(n+1) < p \leq 1$.

## Elliptic operators with unbounded diffusion coefficients in $L^p$ spaces

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**Abstract.** In this talk we focus our attention on elliptic operators with unbounded diffusion of the form

$$Lu = (1 + |x|^\alpha)\Delta u, \tag{1}$$

for positive values of $\alpha$, on $L^p = L^p(R^N, dx)$ with respect to the Lebesgue measure. The case $\alpha \leq 2$ has been already investigated by S. Fornaro and L. Lorenzi who proved that the operator above generates a strongly continuous and analytic semigroup in $L^p$ and in spaces of continuous functions. For $1 < p < \infty$ an explicit description follows from the a-priori estimates

$$\|(1 + |x|^\alpha)D^2 u\|_p \leq C(\|u\|_p + \|(1 + |x|^\alpha)\Delta u\|_p.$$

Similar estimates hold for a mor general class of operators. They can be deduced by some weigthed norm inequalities for Caldéron-Zygmund singular integrals. Muckenhoupt and Wheeden for example proved that estimates of the form

$$\|aD^2u\|_p \leq C\|a\Delta u\|_p$$

are true for weights $a$ in suitable Muckenhoupt classes. In particular the estimates above imply that

$$\||x|^\alpha D^2u\|_p \leq C\||x|^\alpha \Delta u\|_p \tag{2}$$

and

$$\|(1+|x|^\alpha)D^2u\|_p \leq C(\|u\|_p + \|(1+|x|^\alpha)\Delta u\|_p$$

for $0 < \alpha < \frac{N}{p'}$ where $p'$ is the conjugate exponent of $p$.
Similar estimates follow also by the work of Kree who studied singular integrals in $L^p$ spaces with respect to the weight $1+|x|^\alpha$, $-\frac{N}{p'} < \alpha < \frac{N}{p'}$.
We will prove that for $2 < \alpha \leq (N-2)(p-1)$ and $N \geq 3$ the operator above generates a semigroups in $L^p$ which is analytic when for $\alpha < (N-2)(p-1)$. Moreover for $2 < \alpha < N/p'$ an explicit description of the domain follows from an improved version of the a-priori estimates (2).

## Gradient structures for reaction diffusion systems and semiconductor models for interface dynamics

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## On a model in radiation hydrodynamics

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**Abstract.** We consider a simplified model arising in radiation hydrodynamics based on the Navier-Stokes-Fourier system describing the macroscopic fluid motion, and a transport equation modelling the propagation of radiative intensity. It is a joint work with B. Ducomet and E. Feireisl.

References

[1] B. Ducomet, E. Feireisl, Š. Nečasová: On a model in radiation hydrodynamics, Submitted.

## Approximation of Semigroups and Inversion of the Laplace Transform

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**Abstract.** We discuss new error estimates for rational approximations of operator semigroups (i.e., approximations of semigroups in terms of finite sums of the resolvents of their generators) and their applications to Laplace transform inversion.

**Regularity theory for the moving contact line**

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**Abstract.** The spreading of a thin liquid droplet on a solid surface is described by the the so-called thin-film equation. It comes as a fourth order degenerate parabolic equation for the film height $h(t,x)$ as a function of the substrate $x$ and time $t$. For a 1-d substrate it reads

$$\partial_t h + \partial_x(h^n \partial_{xxx} h) \;=\; 0.$$

There is a well-developed theory for weak solutions.

We are interested in the moving contact line (the triple junction between solid, liquid and vapor), described by its position $X(t)$ on the substrate. It leads to a free boundary problem, which in case of $n = 1$ reads

$$h(t,X(t)) = 0, \quad \partial_x h(t,X(t)) = 0, \quad \frac{dX}{dt}(t) \;=\; \partial_{xxx} h(t,X(t))$$

Mathematically speaking, this free boundary problem is a natural generalization to fourth order of the one for the porous medium equation.

We present an long-time existence theory for classical solutions, with initial date which are small perturbations of the stationary solution $h(x) = (\max\{x,0\})^2$. It is based on the maximal regularity of the linearized operator in weighted $L^2$-spaces, a suitable transformation of the nonlinear problem, and the identification of a critical norm for the nonlinear problem. The critical norm is an interpolation norm, which after transformation can be characterized in Fourier space.

**Semigroups and geometric measure theory in Wiener spaces**

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**Abstract.** Functions of bounded variation and sets of finite perimeter in an abstract Wiener space, i.e., an infinite-dimensional Banach space endowed with a Gaussian measure and a related differential structure, have been introduced by M. Fukushima and M. Hino using Dirichlet forms, and their properties have been studied with tools from analysis and stochastics. In the papers quoted below, the theory is presented using only analytical tools, basically Ornstein-Uhlenbeck semigroups and integralgeometric formulae, and the first properties of the boundary of a set of finite perimeter are investigated from the point of view of geometric measure theory. In [4], in the special case of a Hilbert space, another approach is presented, based on a different Ornstein-Uhlenbeck semigroup which enjoys stronger regularising properties.

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### Global weak solutions for reaction-diffusion systems: a mixed $L^1 - L^2$ approach

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**Abstract.** Lots of reaction-diffusion systems arising in applications come with the two natural following properties:
*Positivity of the solutions is preserved for all time.*
*The total mass of the components is controlled for all time*.
The fact that the total mass of the components does not blow up in finite time suggests that solutions should exist for all time (solutions are actually bounded in $L^1$ uniformly in time). But, it turns out that the answer is not so simple. In particular, it is necessary to give up looking for bounded classical solutions and rather consider *weak solutions* which may be allowed to blow up in $L^\infty$ at some time(s), and nevertheless continue to exist. For instance, global weak solutions exist as soon as the nonlinearities are bounded in $L^1$.
A curious $L^2$-estimate is a priori valid for all these systems. This estimate allows to prove global existence when nonlinearities are at most quadratic as it is the case of many chemical and biological systems. Actually, an a priori $L^2$-compactness even holds: it turns out to be an adequate tool to study the limit of chemical systems where some rate constants tend to infinity. We will describe some recent results in this direction together with open problems, after giving an overview of the situation.

### Global solutions to hyperbolic Navier-Stokes equations

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**Abstract.** We consider a hyperbolicly perturbed Navier-Stokes initial value problem in $\mathbb{R}^n$, $n = 2, 3$, arising from using a Cattaneo type relation instead of a Fourier type one in the constitutive equations. The resulting system is an essentially hyperbolic one with quasilinear nonlinearities. The global existence of smooth solutions for small data is proved, and relations to the classical Navier-Stokes systems are discussed. (Joint work with J. Saal)

### On parabolic equations with non-homogeneous Neumann boundary conditions

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**Abstract.** It is well known that the question how to treat parabolic equations with inhomogeneous Neumann conditions in case of nonsmooth data of the problem is a delicate one. There are approaches in the literature where the Banach space under consideration is a negatively indexed Sobolev space of type $H^{-\theta,q}$ or a Sobolev-Morrey space. The serious disadvantage of this, however, is that one obtains at the end that for any time point the divergence of the current is only a distribution. But it would be highly satisfactory to know that the normal flow over any part of the boundary is well defined by Gauss' theorem, because the continuity of the normal component of the current plays an essential role in connecting and embedding of potential flow systems.

This means that in Gauss' theorem

$$\int_\Omega \mu\nabla\varphi\cdot\nabla\psi\,d\mathrm{x} = -\int_\Omega div\big(\mu\nabla\varphi\big)\;\psi\,d\mathrm{x} + \langle\nu\cdot\mu\nabla\varphi, tr(\psi)\rangle_\partial.$$

(suitable) indicator functions should be admissable as test functions $\psi$. Following an idea of K. Gröger, we take $X := L^p(\Omega\cup\Gamma; d\mathrm{x}+\sigma_\Gamma)$ as the corresponding Banach space, where $d\mathrm{x}$ is the Lebesgue measure on $\Omega$ and $\sigma_\Gamma$ is the induced boundary measure on the Neumann boundary part $\Gamma$. This has the advantage, that the distribution $\nu\cdot\mu\nabla\varphi$ is forced to be a $L^p(\Gamma;\sigma_\Gamma)$- function. Our main result is that second order divergence operators with mixed boundary conditions satisfy maximal parabolic regularity on $X$, inclusively consequences for nonlinear equations.

### Stability of advective systems and creeping flows of viscoelastic fluids

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**Abstract.** It is shown that recent results on linear stability of "advective" systems [1] can be applied to give a rigorous foundation for studying the stability of creeping flows of certain viscoelastic fluids. This result requires periodic boundary conditions. To ascertain stability, a bicharacteristic amplitude equation needs to be analyzed in addition to the eigenvalues of the linearized operator. I shall also present a result which allows to infer nonlinear stability from linear stability under more general hypotheses.

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### Markov uniqueness and $L_1$-uniqueness of degenerate elliptic operators

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**Abstract.** Let $\Omega$ be an open subset of $\mathbf{R}^d$ and $H = -\sum_{i,j=1}^d \partial_i\, c_{ij}\,\partial_j$ a second-order partial differential operator on $L_2(\Omega)$ with domain $C_c^\infty(\Omega)$ where the coefficients $c_{ij}\in W^{1,\infty}_{\rm loc}(\overline{\Omega})$ are real symmetric and $C=(c_{ij})$ is a strictly positive-definite matrix over $\Omega$. Let $c$ be a positive function such that $0<C(x)\le c(|x|)I$ for all $x\in\Omega$. Consider the following conditions:

1. $H$ has a unique extension which generates a continuous semigroup on $L_1(\Omega)$,
2. $H$ has a unique extension which generates a submarkovian semigroup on $L_2(\Omega)$,
3. the capacity of $\partial\Omega$ measured with respect to $H$ is zero.

Then 1⇒2⇒3 and if

$$\int_0^\infty ds\, s^{d/2}\, e^{-\lambda\,\mu(s)^2} < \infty$$

for some $\lambda>0$ where $\mu(s)=\int_0^s dt\, c(t)^{-1/2}$ then all three conditions are equivalent.

We will discuss these results and give some examples and applications.

### Uniqueness of Fokker-Planck Equations on Infinite Dimensional Spaces

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**Abstract.** This talk is about the stochastic diffusion equation

$$dX(t) = \mathrm{div}[\mathrm{sgn}(\nabla(X(t)))]dt + \sqrt{Q}dW(t) \text{ in } (0,\infty) \times \mathscr{O}$$

where $\mathscr{O}$ is a bounded open subset of $\mathbb{R}^d$, $d = 1, 2$, $W(t)$ is a cylindrical Wiener process on $L^2(\mathscr{O})$ and $\mathrm{sgn}(\nabla X) = \nabla X/|\nabla X|_d$ if $\nabla W \neq 0$ and $\mathrm{sgn}(0) = \{v \in \mathbb{R}^d : |v|_d \leq 1\}$. The multivalued and highly singular dffusivity term $\mathrm{sgn}(\nabla X)$ describes interaction phenomena and the solution $X = X(t)$ might be viewed as the stochastic flow generated by the gradient of the total variation $\|DX\|$. Our main result says that this problem is well posed in the space of processes with bounded variation in the spatial variable $\xi$. The above equation is relevant for modeling crystal growth as well as for total variation based techniques in image restoration.

### Linearized stability for nonlinear evolution equations

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**Abstract.** We introduce a general notion of 'resolvent differential' for an $\omega-$accretive, possibly multivalued, operator $A \subset X \times X$ in a Banach space $X$, and derive a corresponding principle of linearized stability at an equilibrium point for the Cauchy problem $\dot{u}(t) + Au(t) \ni 0,\ t \geq 0,\ u(0) = u_0$, associated with $A$. The result dispenses with range conditions (on $A$) as in a previous result, and thus allows for a unified treatment of further linearization principles for more general evolution equations of the form $\dot{u}(t) + Au(t) \ni f(u(t)),$ as well as for partial differential delay equations under (just) subtangential existence assumptions.

### Maximal regularity for cylindrical boundary value problems

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**Abstract.** The proof of maximal regularity for general boundary value problems on domains with non-compact boundaries relies typically on intricate localization procedures. In the talk presented, I will show that for the $L^p$-approach such procedures can be avoided, if the domain (and the corresponding differential operator) is cylinder like, i.e., of the form $\Omega = \Omega_1 \times \Omega_2$ and maximal regularity is known on $\Omega_1 \subset \mathbb{R}^k$ and $\Omega_2 \subset \mathbb{R}^m$. For domains of this type there is a much more elegant approach based on the joint $H^\infty$-calculus result of N. Kalton and L. Weis. The approach has also some advantages in comparison to localization procedures. In fact, it allows for an extension of the typical (and sharp) range $(3+\varepsilon)' < p < 3+\varepsilon$ for 3D Lipschitz domains to $(4+\varepsilon)' < p < 4+\varepsilon$, if $\Omega = \Omega_1 \times \Omega_2$ and $\Omega_1$ is a 2D Lipschitz domain. The presented results have been developed in a joint project with Tobias Nau at the University of Konstanz.

### Analysis of the free boundary value problem related to the spin-coating process

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**Abstract.** We consider an incompressible viscous flows in an infinite layer like domain with a free surface and Navier's slip boundary conditions related to the spin-coating process. We establish the existence of a unique solution for arbitrary $T > 0$ in the $L^p$-framework for $p > 5$ provided the data are small enough. Our strategy is to apply the maximal regularity approach to the linearized equations by the method of Newton polygon.

### $L^p$ theory for tangentially degenerate operators

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**Abstract.** We investigate elliptic second order operators with Dirichlet boundary conditions where the diffusion coeffcients decay in first order in the tangential directions. We show that such operators generate consistent analytic semigroups on $L^p$, $C_b$ and $C_0$, and explore their properties. In particular, we give an explicit description of the domains. This paper is joint work with S. Fornaro, G. Metafune and D. Pallara, and complements previous work by my co-authors with J. Prüss on the case of full degeneration in first order.

### A Quasi-stationary Stefan Problem with Surface Tension

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**Abstract.** Let $\Omega_0$ be a bounded domain in $\mathbb{R}^n$ with smooth boundary $\Gamma_0$. We are interested in the solidification/melting process of a liquid filling the area $\Omega_0$ at time $t = 0$. We denote by $\Omega(t)$ the area filled at time $t$ and by $\Gamma(t)$ its boundary.
Modeling leads us to the assumption that $\Omega$ evolves according to certain equations involving the temperature $u = u(x, t)$ in $x$ at time $t$, its (outer) normal derivative $\partial_\nu u$, the normal velocity $V$ of $\Gamma(t)$, and its mean curvature $\kappa$:

$$\begin{aligned} \Delta u &= 0 \quad \text{in } \Omega(t) \\ V + \partial_\nu u &= 0 \quad \text{on } \Gamma(t) \\ \alpha V + \sigma\kappa &= u \quad \text{on } \Gamma(t) \\ \Gamma(0) &= \Gamma_0 \quad \text{at } t = 0. \end{aligned}$$

Here, $\sigma$ is the surface tension coefficient and $\alpha$ the relaxation coefficient. The case $\alpha = 0$ is known as Gibbs-Thomson's law.
We assume here first that $\sigma = 1$, while $\alpha$ is a smooth strictly positive function, and we show the existence of a short time solution. We then consider the situation, where $\alpha(x)$ becomes zero on certain parts of the surrounding space while being strictly positive in others.

**On the Stokes equations with first order boundary condition and its application to the Navier-Stokes equations**

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**Abstract.** In this talk we consider a $L_p$-$L_q$ maximal regularity result and also some decay properties of solutions for the Stokes system with first order boundary condition. Moreover, we discuss its application to the free boundry problem for the one phase Navier-Stokes equations.

**Local well-posedness of incompressible two-phase flow with phase transition**

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**Abstract.** We discuss recent results on local well-posedness of incompressible two-phase flow with phase transition. This system has completely different properties which depends on whether densities are equal or not. The regularity of the height function of the system with equal densities is the same regularity as the height function of the two-phase Stefan problem with Gibbs-Thomson correction. On the other hand, the regularity of the height function of the system with different densities is the same regularity as the two-phase Navier-Stokes problem with surface tension. The part of this work is a joint project with J. Prüss (Halle), Y. Shibata (Waseda), and G. Simonett (Nashville).

**On the Rayleigh-Taylor instability for the two-phase Navier-Stokes equations**

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**Abstract.** We consider the motion of two superposed, immiscible, viscous, incompressible fluids that are separated by a sharp interface which needs to be determined as part of the problem. Allowing for gravity to act on the fluids, we prove local well-posedness of the problem. In particular, we obtain well-posedness for the case where the heavy fluid lies on top of the light one, that is, for the case where the Rayleigh-Taylor instability is present. Additionally we show that solutions become real analytic instantaneously, and we study the Rayleigh-Taylor instability. (Joint work with Jan Prüss.)

**Stochastic Navier-Stokes-Coriolis Equations**

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**Abstract.** We consider the Navier-Stokes equations with Coriolis term with periodic boundary conditions perturbed by a cylindrical Wiener process. Weak and stationary martingale solutions to the associated stochastic evolution equation are constructed. The time-invariant distribution of the stationary martingale solution can be interpreted as the long-time statistics of random fluctuations of the stochastic evolution around the Ekman spiral, which is an explicit stationary solution of the Navier-Stokes equations with Coriolis term. This is the stochastic analogue of the asymptotic stability of the Ekman spiral proven by Hess.

### Partial Gaussian bounds for degenerate differential operators

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**Abstract.** Let $S$ be the semigroup on $L_2(\mathbf{R}^d)$ generated by a degenerate elliptic operator, formally equal to $-\sum \partial_k\, c_{kl}\, \partial_l$, where the coefficients $c_{kl}$ are real bounded measurable and the matrix $C(x) = (c_{kl}(x))$ is symmetric and positive semi-definite for all $x \in \mathbf{R}^d$. Let $\Omega \subset \mathbf{R}^d$ be a bounded Lipschitz domain and $\mu > 0$. Suppose that $C(x) \geq \mu\, I$ for all $x \in \Omega$. We show that the operator $P_\Omega\, S_t\, P_\Omega$ has a kernel satisfying Gaussian bounds and Gaussian Hölder bounds, where $P_\Omega$ is the projection of $L_2(\mathbf{R}^d)$ onto $L_2(\Omega)$.
Similar results are for the operators $u \mapsto \chi\, S_t(\chi\, u)$, where $\chi \in C^\infty_{\rm b}(\mathbf{R}^d)$ and $C(x) \geq \mu I$ for all $x \in \operatorname{supp} \chi$.
This is joint work with El Maati Ouhabaz.

### Global Regularity for the Three-dimensional Primitive Equations of Atmospheric and Oceanic Dynamics

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AND

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**Abstract.** In this talk I will show the global existence and uniqueness of strong solutions to the three-dimensional Primitive Equations of atmospheric and oceanic dynamics.
Inspired by this result I will also provide a new global regularity criterion for the three-dimensional Navier-Stokes equations involving one component of the pressure gradient.

This is a joint work with Chongsheng Cao.

### Stabilization and backward uniqueness of parabolic-hyperbolic fluid structure interaction models

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**Abstract.** The established model consists of a structure (modeled by the hyperbolic system of dynamic elasticity)which is immersed in a fluid (parabolic linearized Navier-Stokes equations) with coupling at the interface between the two media. The following issues will be presented: (i) stabilization - from strong to uniform - the former ( for the original model)depending on the geometry of the structure; the latter, with an additional damping term at the interface, independent on the geometry of the structure; (ii) backward uniqueness of the underlying semigroup. The case of non-linear damping at the interface will be included.

### Identification of the domain in $L^p$ of non-symmetric Ornstein-Uhlenbeck operators

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**Abstract.** Let $(P(t))_{t\geq 0}$ be the Ornstein-Uhlenbeck semigroup associated with the stochastic Cauchy problem

$$dU(t) = AU(t)\,dt + dW(t),$$

where $A$ is the generator of a $C_0$-semigroup $(S(t))_{t\geq 0}$ on a Hilbert space $H$, and $(W(t))_{t\geq 0}$ is an $H$-cylindrical Brownian motion. Assuming that $-A$ admits a bounded $H^\infty$-calculus of angle $< \frac{1}{2}\pi$, we determine the $L^p$-domain of the generator of $P$. An extension to the Banach space setting is discussed as well. The results are applied to the 1D stochastic heat equation driven by additive space-time white noise. This is joint work with Jan Maas.

### Stochastic maximal regularity

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**Abstract.** In this talk we discuss our recent progress on maximal regularity of convolutions with respect to Brownian motion. Under certain conditions, we show that stochastic convolutions

$$\int_0^t S(t-s)f(s)dW(s)$$

satisfy optimal $L^p$-regularity estimates and maximal estimates. Here $S$ is an analytic semigroup on an $L^q$-space. We also provide counterexamples to certain limiting cases and explain the applications to stochastic evolution equations. The results extend and unify various known maximal $L^p$-regularity results from the literature.

References

[1] Jan van Neerven, Mark Veraar, Lutz Weis, Maximal $L^p$-regularity for stochastic convolutions in $L^q$-spaces, preprint.

### A new approach to prove regularity of solutions of degenerate/parabolic equations

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**Abstract.** By using a new technique developped in the farmework of Harnack inequalities by E. DiBenedetto, U. Gianazza and myself, we are able to show an alternative approach with respect to the usual ones.

### Nonlinear parabolic equation with variable exponents and $L^1$ data

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**Abstract.** We prove existence and uniqueness of renormalized solutions to nonlinear parabolic equations with variable exponents for $L^1$-data. The functional setting involves Lebesgue-Sobolev space with variable exponents.

References

[1] P. Wittbold and A. Zimmermann, Existance and uniqueness of renormalized solutions to nonlinear elliptic equations with variable exponents and $L^1$-data, *Nonlinear Analysis TMA 72*, 2990-3008, 2010.

[2] M. Bendamana and P. Wittbold and A. Zimmermann, Nonlinear parabolic equation with variable exponents and $L^1$ data, to appear in J.Diff.Equ. 2010.

### Spectral Theory via operator M-functions - forward and inverse problems

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**Abstract.** A useful tool in studying forward and inverse problems for ODEs is given by the Weyl-Titchmarsh m-function. In PDE problems, a similar role is played by the Dirichlet-to-Neumann map. Both of these can be determined solely from the boundary behaviour of solutions. In this talk, we will look at some results in this area, before focussing on extending m-functions and Dirichlet-to-Neumann maps to the abstract setting of boundary triples, giving rise to operator M-functions. We will discuss properties of M-functions and their relation to the resolvent and the spectrum of the associated operator.

References

[1] B. M. Brown, G. Grubb, I. Wood: $M$-functions for closed extensions of adjoint pairs of operators with applications to elliptic boundary value problems, *Math. Nachr.*, **3**, 314 - 347 (2009).

[2] B. M. Brown, J. Hinchcliffe, M. Marletta, S. Naboko, I. Wood: The abstract Titchmarsh-Weyl $M$-function for adjoint operator pairs and its relation to the spectrum, *Int. Eq. Oper. Th*, **63**, 297 - 320 (2009).

[3] B. M. Brown, M. Marletta, S. Naboko, I. Wood: Boundary triplets and $M$-functions for non-selfadjoint operators, with applications to elliptic PDEs and block operator matrices, *J. London Math. Soc.*, **77**, 700 - 718 (2008).

**The unique existence and the stability of solutions of two-dimensional Navier-Stokes exterior problem with external force with symmetry**

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**Abstract.** This talk concerns the stationary Navier-Stokes equation in a two-dimensional exterior domain under an external force. We assume that the external force is given by some potential functions, and that satisfies antisymmetry conditions, together with the domain, considered in a previous work [1]. The assumption on the potential function is improved to some extent. We also show the stability of the stationary solution under small initial perturbation belonging to a suitable class. This is a joint work with Professor G. P. Galdi (Pittsburgh).

References

[1] M. Yamazaki, The stationary Navier-Stokes equation on the whole plane with external force with antisymmetry, Ann. Univ. Ferrara, **55**, 407–423 (2009).

**On convergence of solutions to equilibria for quasilinear parabolic problems**

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**Abstract.** We show convergence of solutions to equilibria for quasilinear parabolic evolution equations in situations where the set of equilibria is non-discrete, but forms a $C^1$-manifold which is normally stable. To illustrate the scope of the so-called generalized principle of linearized stability we discuss among others convergence to equilibria for Stefan problems with surface tension. This is joint work with Jan Prüss (Halle) and Gieri Simonett (Nashville).