Comparison of Dual Orderings in Time II

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Aims and results
In the direction of researches on formalization in the social sciences [6,1,7], several papers were devoted to analyzing the dual interplay between cultural components (categories of words) and actual practices (welfare treatments, programs ...). A first analysis of poverty in NY City in 1888 - 1917 [6] was undertaken in a joint work, with the description of relief treatments by words to investigate their institutional logic.

By making use of the abilities of lattices to analyze the duality treatments × words, a second note [1] refined this analysis along three directions. First, to screen the source data with the basic toolkit of FCA [10, 4 …] and Lattice Analysis [2] (orders on words, treatments, concept lattices …). Then, to make use of a second tool set for elaborating more synthetic views of the data source structures with canonical basis of implications [5], lattice splits generated by transpositions / double arrows expressing incompatibilities between words / treatments, and lattice ungluing decompositions [4] into intervals that expresses similarities between words or treatments and provides an objective and faithful way for dismantling the ordinal data structure. The third direction compares the findings in 1888 / 1917, and addresses the question of what was either stable, or different between these two points in time through a formal comparison using simple if not simplistic consensus by context union / intersection.

The aim of the present work is to elaborate and experiment new algorithms for pointing out more systematically what is new or unmoved concerning orders and lattice structures, as they change through time (see Fig. 1-2), and to test them on the original data set. To this end, we will mix together and make use of specific / relative basis of implications [3] that naturally occur when apposition and subposition of contexts have to be considered, together with subdirect products of lattices (see Fig. 3) that have been used in particular for context fusion [11], as a natural candidate for lattice consensus. The outcome is to give a simultaneous representation of the two data sets providing new ways to explore and characterize practice / cultural changes.

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Fig. 1. The concept lattice \textit{treatments} × \textit{words} (1888), together with the two canonical basis of implications of implications on conjunctions of \textit{words} (left) and \textit{treatments} (right hand side).

Fig. 2. The concept lattice \textit{treatments} × \textit{words} (1917), with its canonical basis of implications characterizing minimally discrepancy to treatment/word powersets.
As for the results, both *treatment* × *word* lattices (for 1888 and 1917) are quite small as compared with the potential 2ⁿ elements (26 and 18 elements respectively). This reveals a lot of implications between conjunctions of *words* (or *treatments*, dually), which are summarized by their *canonical basis* of implications (see Fig. 1-2, where the lattices are *minimally labeled* [8]). Most of their implications have a *single premise*, which means that these lattices are nearly *distributive*. Actually the intervals above $s$:*shelter* and $i$:*investigation* (in the two lattices, respectively) are distributive. Interestingly, these two lattices are decomposable in *unglued intervals* [4 §5.2] which assesses similarities [1] between *words* (*treatments*) respecting the global structure.

1888 reveals the *splits* treatment / word (transpositions expressed by *double arrows* in the contexts see [4]): paidWork / NEEDY, investigation / DESTITUTE, advise / INDIGENT, findJob / WORTHY, give$ / HOMELESS, food / FALLEN, and asylum / STRANGER. As shocking as it could appear now, in 1888 one gave asylum - except to strangers! -, or money -except to HOMELESS! - etc. Similarly, 1917 displays the splits: jobTrain / NEEDY, food-shelter-asylum / DISTRESSED, advise / WORTHY and give$ / HOMELESS as it was already the case in 1888. As local negations, these splits capture fundamental distinctions in systems of moral boundaries of these times.

Now a first natural idea for comparing these two lattices is to glue their contexts horizontally by taking their *apposition* (resp. vertically *subposition*), and to construct the corresponding lattice which is *join-embedded* (resp. *meet*) in their direct product, as it is implicitly done with *nested line diagrams* [10], and to distinguish two *specific basis* [1] of implications going from one set to another (ex: 1888 ↔ 1917 words).

![Fig. 3. The fusion of the 1888 & 1917 lattices is embedded in their direct product and is also gluing decomposable. The two relative basis of implications (1888 / 1917 = lower / upper-case letters) express discrepancy to direct product and independence.](image-url)
This is specially adapted when a single set of objects is described through two different sets of attributes with a dissymmetry between object / attribute rôles. In our present case however, there is a symmetry words / treatments which are equally conceptual. On the other hand two pairs of different sets are needed to distinguish them for these two periods. Hence, let \((T_1, W_1, I_1)\) and \((T_2, W_2, I_2)\) be the 1888 / 1917 contexts and \(L_1 = L(T_1, W_1, I_1)\), \(L_2 = L(T_2, W_2, I_2)\) their concept lattices. The fusion (see \([11, 4 \text{ §5.1}]\)) of these contexts is the context generating the smallest sublattice of \(L_1 \times L_2\) the relation of which being a superset of the relation obtained by subposition of the two appositions \((I_1 \mid I_1 \cup I_2)\), and \((I_1 \cup I_2 \mid I_1)\). This subdirect product construction is highly symmetric regarding the two original contexts, as well as words & treatments. The two relative basis of implications mixed together characterize minimally the discrepancy to direct product (taken as a starting lattice \([3,9]\) or as background knowledge \([8]\)), and the underlying meet / join morphisms between factors. After implementation through GLAD \([2]\), the resulting lattice (see Fig. 3) appears to be gluing decomposable, which allows detecting attributes that are structurally similar (inn:misfortune / G:deserving,...) or stable (distressed, stranger, fallen...) in time, which now requires careful screenings and further interpretations.

References