

# Comparison of Dual Orderings in Time II

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## Aims and results

In the direction of researches on formalization in the social sciences [6,1,7], several papers were devoted to analyzing the dual interplay between cultural components (categories of words) and actual practices (welfare treatments, programs ...). A first analysis of *poverty in NY City in 1888 - 1917* [6] was undertaken in a joint work, with the description of *relief treatments by words* to investigate their institutional logic.

By making use of the abilities of lattices to analyze the duality *treatments  $\times$  words*, a second note [1] refined this analysis along three directions. First, to screen the source data with the basic toolkit of FCA [10, 4 ...] and Lattice Analysis [2] (*orders* on words, treatments, *concept lattices* ...). Then, to make use of a second tool set for elaborating more synthetic views of the data source structures with *canonical basis of implications* [5], lattice splits generated by *transpositions / double arrows* expressing incompatibilities between words / treatments, and lattice *ungluing decompositions* [4] into intervals that expresses similarities between words or treatments and provides an objective and faithful way for dismantling the ordinal data structure. The third direction compares the findings in 1888 / 1917, and addresses the question of what was either stable, or different between these two points in time through a formal comparison using simple if not simplistic consensus by context union / intersection.

The aim of the present work is to elaborate and experiment new algorithms for pointing out more systematically what is new or unmoved concerning orders and lattice structures, as they change through time (see Fig. 1-2), and to test them on the original data set. To this end, we will mix together and make use of *specific / relative* basis of implications [3] that naturally occur when *apposition* and *subposition* of contexts have to be considered, together with *subdirect products* of lattices (see Fig. 3) that have been used in particular for *context fusion* [11], as a natural candidate for lattice consensus. The outcome is to give a simultaneous representation of the two data sets providing new ways to explore and characterize practice / cultural changes.

Program GLAD (C) 1992 V.Duquenne Paris.

Poverty in NY 1888

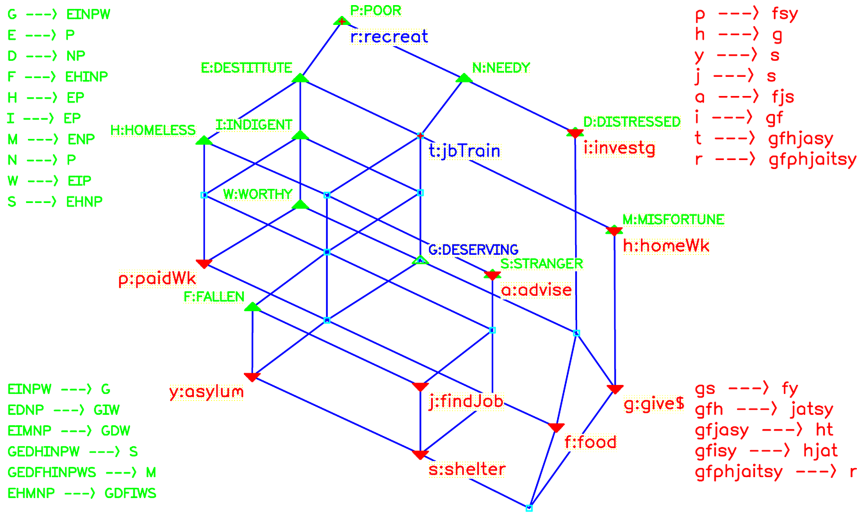


Fig. 1. The concept lattice *treatments* × *words* (1888), together with the two canonical basis of implications of implications on conjunctions of *words* (left) and *treatments* (right hand side)

Program GLAD (C) 1992 V.Duquenne Paris.

Poverty in NY 1917

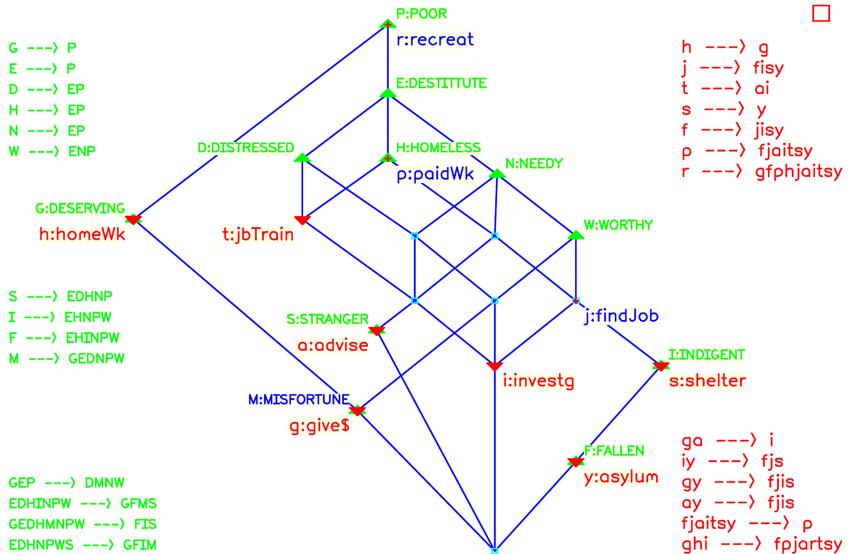
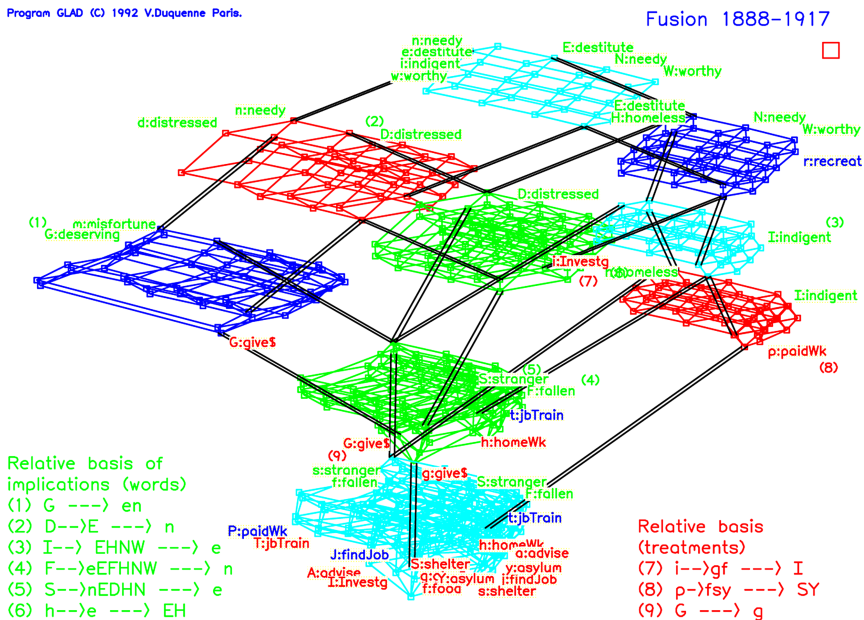


Fig. 2. The concept lattice *treatments* × *words* (1917), with its canonical basis of implications characterizing minimally discrepancy to treatment / word powersets

As for the results, both *treatment* × *word* lattices (for 1888 and 1917) are quite small as compared with the potential  $2^{11}$  elements (26 and 18 elements respectively). This reveals a lot of implications between conjunctions of words (or *treatments*, dually), which are summarized by their *canonical basis* of implications (see Fig. 1-2, where the lattices are *minimally labeled* [8]). Most of their implications have a *single premise*, which means that these lattices are nearly *distributive*. Actually the intervals above *s:shelter* and *i:investigation* (in the two lattices, respectively) are distributive. Interestingly, these two lattices are decomposable in *unglued intervals* [4 §5.2] which assesses similarities [1] between words (*/ treatments*) respecting the global structure.

1888 reveals the *splits treatment / word* (*transpositions* expressed by *double arrows* in the contexts see [4]): paidWork / NEEDY, investigation / DESTITUTE, advise / INDIGENT, findJob / WORTHY, give\$ / HOMELESS, food / FALLEN, and asylum / STRANGER. As shocking as it could appear now, in 1888 one gave asylum - except to strangers!-, or money -except to HOMELESS!- etc. Similarly, 1917 displays the splits: jobTrain / NEEDY, food-shelter-asylum / DISTRESSED, advise / WORTHY and give\$ / HOMELESS as it was already the case in 1888. As local negations, these splits capture fundamental distinctions in systems of moral boundaries of these times.

Now a first natural idea for comparing these two lattices is to glue their contexts horizontally by taking their *apposition* (resp. vertically *subposition*), and to construct the corresponding lattice which is *join-embedded* (resp. *meet-*) in their *direct product*, as it is implicitly done with *nested line diagrams* [10], and to distinguish two *specific basis* [1] of implications going from one set to another (ex: 1888 ↔ 1917 words).



**Fig. 3.** The *fusion* of the 1888 & 1917 lattices is embedded in their *direct product* and is also *gluing decomposable*. The two *relative basis* of implications (1888 / 1917 = lower / upper-case letters) express discrepancy to direct product and independence.

This is specially adapted when a single set of objects is described through two different sets of attributes with a dissymmetry between object / attribute rôles. In our present case however, there is a symmetry *words / treatments* which are equally conceptual. On the other hand two pairs of different sets are needed to distinguish them for these two periods. Hence, let  $(T_1, W_1, I_1)$  and  $(T_2, W_2, I_2)$  be the 1888 / 1917 contexts and  $L_1 = L(T_1, W_1, I_1)$ ,  $L_2 = L(T_2, W_2, I_2)$  their concept lattices. The *fusion* (see [11, 4 §5.1]) of these contexts is the context generating the smallest *sublattice* of  $L_1 \times L_2$  the relation of which being a superset of the relation obtained by subposition of the two appositions  $(I_1 \mid I_1 \cup I_2)$ , and  $(I_1 \cup I_2 \mid I_2)$ . This *subdirect product* construction is highly symmetric regarding the two original contexts, as well as *words & treatments*. The two *relative basis* of implications mixed together characterize minimally the discrepancy to direct product (taken as a *starting lattice* [3,9] or as *background knowledge* [8]), and the underlying *meet / join morphisms* between factors. After implementation through GLAD [2], the resulting lattice (see Fig. 3) appears to be gluing decomposable, which allows detecting attributes that are structurally similar (*m:misfortune / G:deserving,...*) or stable (*distressed, stranger, fallen...*) in time, which now requires careful screenings and further interpretations.

## References

- [1] Duquenne, V., Mohr, J., Le Pape, A.: Comparison of dual ordering in time. Soc. Sci. Information 37, 227–253 (1998)
- [2] Duquenne, V.: Latticial structures in Data Analysis, ORDAL 96: Order and decision-making (I. Rival ed.). Theoretical Computer Science 217, 407–436 (1999)
- [3] Duquenne, V., et al.: Structuration of phenotypes / genotypes through Galois lattices and implications. Applied Artificial Intelligence 17, 243–256 (2003)
- [4] Ganter, B., Wille, R.: Formal Concept Analysis, Mathematical Foundations. Springer, Berlin (1999)
- [5] Guigues, J.L., Duquenne, V.: Familles minimales d'implications informatives résultant d'un tableau de données binaires. Mathématiques & Sciences Humaines 95, 5–18 (1986) (preprint Groupe Mathématiques et Psychologie, Université Paris V-René Descartes (1984))
- [6] Mohr, J., Duquenne, V.: The duality of culture and practice: Poverty relief in New-York City, 1888-1917. Theory and Society 26, 305–356 (1997)
- [7] Mohr, J., Bourgeois, M., Duquenne, V.: The Logic of Opportunity: A Formal Analysis of the University of California's Outreach and Diversity Discourse. Center for Studies in Higher Education, UC Berkeley, Research and Occasional Papers Series (2004), <http://csh.berkeley.edu/publications/papers/papers.html>
- [8] Stumme, G.: Attribute exploration with background implications and exceptions. In: Bock, H.H., Polasek, W. (eds.) Data Analysis and Information Systems. Statistical and Conceptual approaches. Studies in classification, data analysis and knowledge organization, vol. 7, pp. 457–469. Springer, Heidelberg (1996)
- [9] Valtchev, P., Duquenne, V.: Towards scalable divide-and-conquer methods for computing concepts and implications. In: SanJuan, E., et al. (eds.) Proc. of the Journées de l'Informatique Messine (JIM 2003): Knowledge Discovery and Discrete Mathematics, Metz (FR), September 3-6, 2003, pp. 3–14. INRIA (2003)
- [10] Wille, R.: Restructuring lattice theory: an approach based on hierarchies of concepts. In: Rival, I. (ed.) Ordered sets, pp. 445–470. Reidel, Dordrecht, Boston (1982)
- [11] Wille, R.: Sur la fusion des contextes individuels. Mathématiques & Sciences Humaines 85, 57–71 (1984)