

course-fellner

The short course will discuss various ideas of entropy- and duality estimates in the theory of global solutions of reaction-diffusion type systems. Going back to ideas of Boltzmann and Grad in the context of gas dynamics, the so-called entropy method considers suitable convex Lyapunov functionals and aims to quantify the monotone entropy decay in terms of a functional inequality between entropy dissipation rate and an entropy functional relative to an entropy minimising equilibrium. By being based on functional inequalities (rather than direct estimates on solutions) the entropy method is quite robust with respect to model variations and generalisation. Moreover, the entropy method is precisely a nonlinear approach, which usually allow to estimate explicitly the involved constants. We will show how the entropy method allows to prove exponential convergence to equilibrium with explicit rates and constants for reaction-diffusion systems (and also coagulation-fragmentation models) as long as a solution with well-defined entropy exists. Moreover, we shall demonstrate that the entropy method does not only apply to detailed balance systems, but also to more general systems like linear complex balance systems and even more general first order reaction networks. The latter is a fine example for the robustness of the entropy method. Finally, we will present a recently improved duality approach, which shows, for instance, global existence of weak solutions for reversible reaction-diffusion systems with quadratic reaction terms in 2D.