WASEDA University

TECHNISCHE UNIVERSITÄT DARMSTADT

THE UNIVERSITY OF TOKYO

International Research Training Group 1529

Mathematical Fluid Dynamics

Japanese-German International Workshop on Mathematical Fluid Dynamics

Darmstadt, Germany
November 30 – December 2, 2016

Lecture Series

Volker Betz, Darmstadt
Rough paths and rough differential equations

Takayoshi Ogawa, Sendai
Global behavior of solutions to a drift-diffusion system

Marius Paicu, Bordeaux
Topics in mathematical analysis of some complex fluids

Confirmed Participants

Didier Bresch, Le Bourget du Lac
Raphaël Danchin, Paris
Eduard Feireisl, Prag
Toshiaki Hishida, Nagoya
Hans Knüpfer, Heidelberg
Mads Kyed, Darmstadt
Paolo Maremonti, Neapel
Piotr Mucha, Warschau
Šárka Nečasová, Prag
Tohru Ozawa, Tokyo
Jan Prüß, Halle
Jürgen Saal, Düsseldorf
Jonas Sauer, Leipzig
Christian Seis, Bonn
Gieri Simonett, Nashville
Ryo Takada, Sendai
Werner Varnhorn, Kassel
Mathias Wilke, Regensburg
Ewelina Zatorska, London

Organizers

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Wednesday, 30. November 2016

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09:50-10:35	Volker Betz	*Rough paths and rough differential equations*
11:00-11:20	Patrick Tolksdorf	*Recent results on the Stokes and Navier-Stokes equations on bounded Lipschitz domains*
11:25-11:55	Jan Prüss	*Critical Spaces for Semilinear Parabolic Evolution Equations*
12:00-12:30	Mathias Wilke	*Critical spaces for the Navier-Stokes equations with Navier boundary conditions*
14:00-14:30	Eduard Feireisl	*Measure-valued solutions in fluid mechanics: Present and Future*
14:35-15:05	Piotr Mucha	*Inhomogeneous Navier-Stokes system in exterior domains in critical functional framework*
15:10-15:30	Matthias Köhne	*Strong Well-Posedness for a Class of Dynamic Outflow Boundary Conditions for Incompressible Newtonian Flows*
15:50-16:00	Go Takahashi	*On partial regularity and extension of solutions to the Navier-Stokes equations*
16:00-16:10	Sebastian Zaigler	*Regularity structures for the primitive equations*
16:10-16:20	Tomoya Kemmochi	*Discretization of maximal regularity and its application to the finite element method*
16:20-16:30	Tobias Seitz	*Enhancement of flow measurements using fluid-dynamic constraints*
16:30-16:40	Krisztian Benyo	*Fluid–structure interaction for the water waves problem*
16:40-16:50	Shota Enomoto	*Large time behavior of solutions to compressible Navier-Stokes equation around space-time periodic solution*
16:50-17:00	Mathis Gries	*A maximum regularity approach to the free boundary value problem for the primitive equations of the ocean*

17:00-17:10	Taiga Kumagai	*A perturbation problem involving singular perturbations of domains for Hamilton-Jacobi equations*
17:20-17:50	Paolo Maremonti	*Singular p-laplacian parabolic problem in exterior domains: higher regularity of solutions and related properties of extinction and asymptotic behavior in time*
17:55-18:25	Ryo Takada	*Global solutions for the incompressible rotating stably stratified fluids*

Thursday, 1. December 2016

Time	Speaker	*Title of Talk*
09:00-09:45	Volker Betz	*Rough paths and rough differential equations*
09:50-10:35	Takayoshi Ogawa	*Global behavior of solutions to a drift-diffusion system*
11:00-11:20	Amru Hussein	*Strong L^p Well-Posedness of the 3D Primitive Equations*
11:25-11:55	Tohru Ozawa	*Remarks on the Rellich inequality*
12:00-12:30	Takahiro Okabe	*Periodic strong solution of Navier-Stokes equations*
14:00-14:30	Hans Knüpfer	*Self-similar lifting and persistent touch-down point solutions in the thin-film equation*
14:35-15:05	Christian Seis	*Stability estimates for continuity equations*
15:10-15:30	Camilla Nobili	*TBA*
15:50-16:00	Naoto Kajiwara	*On a resolvent estimate for bidomain operators*
16:00-16:10	Alexander Dalinger	*On the hydrodynamic limit and equilibrium fluctuations of a particle system with nearest neighbor interactions*
16:10-16:20	Suma'inna	*The existence of $\mathcal{R}$-bounded Solution Operators of The Thermoelastic Plate Equation With Dirichlet Boundary Conditions*

16:20-16:30	Anna Abbatiello	*$p(x)$-Stokes IBVP: ill posedness of uniqueness backward in time*
16:30-16:40	Yuki Kaneko	*TBA*
16:40-16:50	Tobias Tolle	*Extending a hybrid Level Set / Front Tracking method for the simulation of surface tension driven flows*
16:50-17:00	Felix Hummel	*TBA*
17:00-17:10	Anton Seyfert	*Time-Periodic Non-Autonomous Evolution Equations in Interpolation Spaces*
17:20-17:50	Šárka Nečasová	*The motion of a rigid body and a viscous fluid*
17:55-18:25	Tsukasa Iwabuchi	*Besov spaces generated by the Dirichlet Laplacian*

Friday, 2. December 2016

Time	Speaker	*Title of Talk*
09:00-09:45	Takayoshi Ogawa	*Global behavior of solutions to a drift-diffusion system*
09:50-10:35	Volker Betz	*Rough paths and rough differential equations*
10:50-11:10	Hirokazu Saito	*Compressible fluid model of Korteweg type with free boundary condition: model problem*
11:15-11:45	Ewelina Zatorska	*Two-phase model of crowd propagation*
11:50-12:10	Martin Saal	*The Primitive Equations with Linearly Growing Initial Data*
12:20-12:50	Toshiaki Hishida	*L^q-L^r decay estimate of the evolution operator generated by the non-autonomous Oseen operator arising from fluid motion past a rotating obstacle, with applications to the Navier-Stokes initial value problem in 3D exterior domains*
12:55-13:25	Raphaël Danchin	*Optimal time-decay estimates for the compressible Navier-Stokes equations in critical regularity*

$p(x)$-Stokes IBVP: ill posedness of uniqueness backward in time.

Anna Abbatiello

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It is well known that the uniqueness backward in time for smooth solutions to the heat equation is an improperly posed problem. The result is due to Payne in [1]. On the other hand, for the p-laplacian parabolic problem, $p \in (1,2)$, the extinction of the solution in a finite interval of time is one of the major property. In the note [2] we investigate on the extinction property in the case of an electrorheological fluid. In the L^2-setting of weak solutions we are able to prove the following characterization:

there is extinction of a solution to $p(x)$-Stokes IBVP iff $p(x) < 2$.

This is a joint work with F. Crispo and P. Maremonti.

[1] L.E. Payne, *Improperly Posed Problems in Partial Differential Equations*, SIAM, (1975).

[2] A.Abbatiello, F.Crispo and P.Maremonti, *Electrorheological fluids: ill posedness of uniqueness backward in time*, forthcoming.

Fluid–structure interaction for the water waves problem

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In this short talk, first of all present some alternative formulations of the well-known Euler equations (also known as the Water Waves equations) shall be presented, with some of the most pertinent results on existence, well-posedness and the relevant asymptotic regimes. Under the tutelage of Prof. David Lannes of the University of Bordeaux, who is one of the current experts of the domain, my ongoing Ph.D. thesis focuses on the characterisation of the water waves problem coupled with a moving solid object in the fluid domain.

The main objective of the speech will be to present some problematics of such coupled systems as well as to propose possible solutions in the topic which are still being developed. With a big emphasis on the so-called added mass effect, two particular cases will be presented: the problem of a (free) floating structure, and the problem of a moving solid on the bottom of the fluid region.

On the hydrodynamic limit and equilibrium fluctuations of a particle system with nearest neighbor interactions

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For a one-dimensional particle system with nearest neighbor interactions we study the particle density. Starting with a finite number of particles, we rescale time and space and increase the number of particles to infinity. This is called hydrodynamic limit. We show that the particle density converges in the hydrodynamic limit to a solution of a nonlinear heat equation. Furthermore, we consider the fluctuations around its average state, the equilibrium fluctuations.

Optimal time-decay estimates for the compressible Navier-Stokes equations in critical regularity

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The global existence issue for the isentropic compressible Navier-Stokes equations in the critical regularity framework has been addressed in [1] more than fifteen years ago. However, whether (optimal) time-decay rates could be shown in general critical spaces and any dimension $d \geq 2$ has remained an open question. In this joint work with J. Xu, we give a positive answer to that issue not only in the L^2 critical framework of [1] but also in the more general L^p critical framework of e.g. [2]: we show that under a mild additional decay assumption that is satisfied if the low frequencies of the initial data are in e.g. $L^{p/2}(\mathbb{R}^d)$, the L^p norm of the critical global solutions decays like $t^{-d(\frac{1}{p}-\frac{1}{4})}$ for $t \to +\infty$, exactly as firstly observed by A. Matsumura and T. Nishida in [3] in the case $p = 2$ and $d = 3$, for solutions with high Sobolev regularity.

Our method relies on refined time weighted inequalities in the Fourier space, and is likely to be efficient for other hyperbolic/parabolic systems that are encountered in fluid mechanics or mathematical physics.

[1] R. Danchin: Global existence in critical spaces for compressible Navier-Stokes equations,

Inventiones Mathematicae, **141**(3), 579–614 (2000).

[2] F. Charve and R. Danchin: A global existence result for the compressible Navier-Stokes equations in the critical L^p framework,

Arch. for Rat. Mech. and Analysis, **198**(1), 233–271 (2010).

[3] A. Matsumura and T.Nishida: The initial value problem for the equation of motion of compressible viscous and heat-conductive fluids,

Proc. Jpn. Acad. Ser-A, **55**, 337–342 (1979).

Large time behavior of solutions to compressible Navier-Stokes equation around space-time periodic solution

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We consider the compressible Navier-Stokes equation around space-time periodic solution in an infinite layer of $\mathbb{R}^n$ $(n = 2, 3)$ under the action of a space-time periodic external force. If the external force is sufficiently small, then the compressible Navier-Stokes system has a space-time periodic solution. We show that the space-time periodic solution is asymptotically stable under the sufficiently small initial perturbation. Furthermore, it is shown that the asymptotic leading part of the perturbation is given by a product of a solution of the one-dimensional viscous Burgers equation and a space-time periodic function when $n = 2$, and by a product of a solution of the two-dimensional heat equation and a space-time periodic function when $n = 3$.

This talk is based on a joint work with Prof. Y. Kagei of Kyushu University and Mr. M. N. Azlan.

Measure-valued solutions in fluid mechanics: Present and Future

Eduard Feireisl

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In view of some recent results concerning the Euler system, the measure valued solutions reappeared as a suitable alternative to weak solutions for problems in fluid mechanics. We introduce the concept of dissipative measure-valued solutions to both Euler and the Navier-Stokes system and show some applications: Weak strong uniqueness and stability. Finally, some examples of numerical schemes will be given, where this technique leads to positive convergence results.

On partial regularity and extension of solutions to the Navier-Stokes equations

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In this talk, we work on the local-in-time classical solutions to the incompressible Navier-Stokes equations on $\mathbb{R}^n \times (0, T)$. The dimension n is 3 or 4. The objective of this talk is to investigate whether the local-in-time solutions blow up at $t = T$ or can be continued beyond $t = T$. This problem has been treated by several authors. Now we take a slightly different viewpoint and pay attention to the partial regularity of the solutions. First we discuss the partial regularity of solutions, which is an essential argument to obtain the local boundedness of the solutions. Then we will establish a time-extension criterion to the local-in-time solutions as an application of an ϵ-regularity theorem.

A maximum regularity approach to the free boundary value problem for the primitive equations of the ocean

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We report on an ongoing work in progress. The model for the free boundary value problem for the primitive equations of the ocean is presented and the equations are then transformed to yield a set equations on a fixed domain. We solve the linearised problem via the method of optimal $L^p - L^q$-estimates for parabolic problems and then introduce a fixed-point argument for the quasilinear problem.

L^q-L^r decay estimate of the evolution operator generated by the non-autonomous Oseen operator arising from fluid motion past a rotating obstacle, with applications to the Navier-Stokes initial value problem in 3D exterior domains

Toshiaki Hishida

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Consider the motion of a viscous incompressible fluid in 3D exterior domains when a rigid body moves with time-dependent translational and angular velocities. For the linearized system, the L^q-L^r smoothing action near the initial time as well as generation of the evolution operator, which provides a solution to the initial value problem, was shown by Hansel and Rhandi under reasonable conditions. In this presentation we develop the L^q-L^r decay properties for large time of the evolution operator and then apply them to the Navier-Stokes initial value problem.

Strong L^p Well-Posedness of the 3D Primitive Equations

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Primitive equations are considered to be a fundamental model for geophysical flows. Here, the L^p theory for the primitive equations is discussed. This set of equations is globally strongly well-posed for arbitrary large initial data lying in subspaces of $H^{2/p,p}$, $1 < p < \infty$, satisfying certain boundary conditions. Thus, the general L^p setting admits rougher data than the usual L^2 theory with initial data in H^1. In this study, the linearized Stokes type problem plays a prominent role, and it turns out that it can be treated efficiently using perturbation methods for H^∞-calculus.

Besov spaces generated by the Dirichlet Laplacian

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Let Ω be an arbitrary open set of $\mathbb{R}^n$, $n \geq 1$. We consider the definition of Besov spaces on Ω with the Dirichlet boundary condition. We give a definition based on the spectral theorem for the Dirichlet Laplacian. Duality, embedding and lifting properties are also shown.

On a resolvent estimate for bidomain operators

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We study the bidomain equations that are commonly used as a model to represent the electrophysiological wave propagation in the heart. We transform the equations

$$\begin{cases} \partial_t u - \nabla\cdot(\sigma_i \nabla u_i) = 0 & \text{in } (0,\infty)\times\Omega \\ \partial_t u + \nabla\cdot(\sigma_e \nabla u_e) = 0 & \text{in } (0,\infty)\times\Omega \\ u = u_i - u_e & \text{in } (0,\infty)\times\Omega \\ \sigma_i \nabla u_i \cdot n = 0,\ \sigma_e \nabla u_e \cdot n = 0 & \text{on } (0,\infty)\times\partial\Omega \\ u(0) = u_0 & \text{in } \Omega \end{cases}$$

into $\partial_t u + Au = 0$, $u_e = -(A_i + A_e)^{-1} A_i u$, where $A_{i,e} = -\nabla\cdot(\sigma_{i,e}\nabla)$ and $A = A_i(A_i + A_e)^{-1}A_e = (A_i^{-1} + A_e^{-1})^{-1}$(harmonic mean of two elliptic operator). We call A the bidomain operator. In a previous work, the bidomain operator in L^2 space was shown the non-negative self-adjoint operator. In this talk, we show an L^∞ resolvent estimate by a blow-up method and get L^p resolvent estimates ($1 < p < \infty$) by the interpolation and the duality.
This talk is based on a joint work with Prof. Yoshikazu Giga (The University of Tokyo).

Discretization of maximal regularity and its application to the finite element method

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Maximal regularity is one the most important concepts in the theory of nonlinear evolution equations. It is thus natural to ask whether the discrete analogue of maximal regularity holds, when the problem is discretized for numerical computation. In this talk, we consider the full discretization of maximal regularity for the heat problems. We discretize the heat equations by the finite difference method with respect to the time variable, and by the finite element method with respect to the space variables, respectively. Then, we present the discrete version of maximal regularity estimate. We apply this result to the error estimate of the finite element full discretization of the linear and semilinear heat equations. In the error estimate, discrete maximal regularity and the fractional powers of discrete Laplacian play important roles. This talk is based on a joint work with Professor Norikazu Saito (the University of Tokyo).

Self-similar lifting and persistent touch-down point solutions in the thin-film equation

Hans Knüpfer

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In the talk I discuss the appearance of self-similar blow-up solutions for thin-film equations with different mobility exponents. This is related to non-uniqueness phenomena for weak solution of the same equation. The proof is based on dynamical systems arguments.

Strong Well-Posedness for a Class of Dynamic Outflow Boundary Conditions for Incompressible Newtonian Flows

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We derive a class of dynamic outflow boundary conditions for the incompressible Navier-Stokes equations, containing the well-known convective boundary condition but incorporating also the stress at the outlet. We consider the Stokes equations with such dynamic outflow boundary conditions in a halfspace and prove the existence of a strong solution in the appropriate Sobolev-Slobodeckij-setting with L_p (in time and space) as the base space for the momentum balance. For nonvanishing stress contribution in the boundary condition, the problem is actually shown to have L_p-maximal regularity under the natural compatibility conditions.

Aiming at an existence theory for problems in weakly singular domains, where different boundary conditions apply on different parts of the boundary such that these surfaces meet orthogonally, we also consider the prototype domain of a wedge with opening angle $\frac{\pi}{2}$ and different combinations of boundary conditions. Again, maximal regularity of the problem is obtained in the appropriate functional analytic setting and with the natural compatibility conditions.

This is joint work with Dieter Bothe and Takahito Kashiwabara

A perturbation problem involving singular perturbations of domains for Hamilton-Jacobi equations

Taiga Kumagai

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We consider the problem

$$\begin{cases} u^\varepsilon - \dfrac{b \cdot Du^\varepsilon}{\varepsilon} + |Du^\varepsilon| = f & \text{in } \Omega, \\ u^\varepsilon = 0 & \text{on } \partial\Omega, \end{cases}$$

where ε is a positive parameter, Ω is an open subset of $\mathbb{R}^2$ determined through a Hamiltonian function H, $u^\varepsilon : \overline{\Omega} \to \mathbb{R}$ denotes the unknown function, $f : \overline{\Omega} \to \mathbb{R}$ is a given, continuous, and nonnegative function, and $b : \mathbb{R}^2 \to \mathbb{R}^2$ is a Hamiltonian vector field.

We study the asymptotic behavior of solutions u^ε as ε goes to zero. The limit of the solutions is described as solutions of a system of ODEs on a graph. Freidlin-Wentzel, Freidlin-Weber, Sowers, by probabilistic techniques, and Ishii-Souganidis, by PDE techniques, studied stochastic perturbation problems for Hamiltonian flows. Our problem can be seen as a deterministic control version of such perturbation problems.

Singular p-laplacian parabolic problem in exterior domains: higher regularity of solutions and related properties of extinction and asymptotic behavior in time

Paolo Maremonti

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We investigate on the regularity properties of the solutions to the IBVP in exterior (*smooth*) domains of the singular p-laplacian parabolic system. Moreover, we study the behavior of the solution in time. Hence we discuss the important property of the extinction of the solutions and when this does not take place the asymptotic behavior in time. This is a joint work with F. Crispo and C.R. Grisanti.

Inhomogeneous Navier-Stokes system in exterior domains in critical functional framework.

Piotr Mucha

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I plan to talk about a result for the incompressible Navier-Stokes system in an exterior domain. The main goal is to obtain regularity in space of type $L_1(0,T;\dot{B}^s_{p,1}(\Omega))$ in critical framework with respect to the nonlinearity. The main challenge is the result for the Stokes system. The chosen regularity fits very well to needs of the Lagrangian coordinates, then naturally we are able to consider the system with variable density. The talk will based on results from monograph: *Critical functional framework and maximal regularity in action on systems of incompressible flows.* Mem. Soc. Math. Fr. (N.S.) No. 143 (2015), joint with Raphaël Danchin (Paris).

The motion of a rigid body and a viscous fluid

Šárka Nečasová

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We shall consider the problem of the motion of a rigid body in an incompressible viscous fluid filling a bounded domain. We consider the Navier condition on the boundary of the body and the non-slip condition on the boundary of the domain. We prove the global existence of weak solution of the problem. This is a joint work with Nikolai V. Chemetov.

Periodic strong solution of Navier-Stokes equations

Takahiro Okabe

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In this talk, we consider time periodic problem of the incompressible Navier-Stokes equations in the whole space $\mathbb{R}^n$, $n \geq 3$. To construct a time periodic mild solution in the class $BC(\mathbb{R}; L^{n,\infty}(\mathbb{R}^n))$, the class of external forces $BC(\mathbb{R}; L^{n/3,1}(\mathbb{R}^n))$ is critical under the following integral equation:

$$u(t) = \int_{-\infty}^{t} \mathbb{P}e^{(t-s)\Delta} f(s)\, ds - \int_{-\infty}^{t} \nabla \cdot e^{(t-s)\Delta}\mathbb{P}(u \otimes u)(s)\, ds, \qquad t \in \mathbb{R}. \qquad \text{(I.E.)}$$

We study the existence of mild solutions of (I.E.). Then we discuss the strong solvability of the time periodic problem to the Navier-Stokes equations in $L^{n,\infty}(\mathbb{R}^n)$, assuming slight restriction of f in $L^{n,\infty}(\mathbb{R}^n)$.

This talk is based on the joint work with Professor Yohei Tsutsui.

Remarks on the Rellich inequality

Tohru Ozawa

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This talk is based on my recent joint-work with Shuji Machihara and Hidemitsu Wadade. We study the Rellich inequalities in the framework of equalities. We present equalities which imply the Rellich inequalities by dropping remainders. This provides a simple and direct understanding of the Rellich inequalities as well as the nonexistence of nontrivial extremisers.

Critical Spaces for Semilinear Parabolic Evolution Equations

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In this talk, we introduce critical spaces for a general class of semilinear parabolic evolution equations. We prove local well-posedness for large data, as well as global existence for small initial data, taken from the critical spaces These results have applications to quasi-geostrophic equations, vorticity equations, and many other problems. Here we employ the results to study critical spaces for the Navier-Stokes equations with no-slip boundary conditions.

The Primitive Equations with Linearly Growing Initial Data

Martin Saal

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The $3D$ primitive equations are a version of the Navier-Stokes equations used to describe oceanic and atmospheric flows introduced by Lions, Teman and Wang.

The velocity U of the fluid is described by $U = (V, W)$, where $V = (V_1, V_2)$ denotes the horizontal component and W the vertical one, and the surface pressure is Π_s. The primitive equations read

$$\begin{cases} \partial_t V + V \cdot \nabla_H V + W \cdot \partial_z V - \Delta V + \nabla_H \Pi_s &= 0, \\ \mathrm{div}_H V &= -W_z, \\ \partial_z \Pi_s &= 0. \end{cases} \tag{PE}$$

Here ∇_H and div_H denote the horizontal gradient and divergence. Investigating these equations on a layer with data which are linearly growing in the horizontal direction leads to an Ornstein-Uhlenbeck type model, which was treated by Hieber and Sawada for the standard Navier-Stokes equations. In our talk, we linearize the system (PE) and prove the well-posedness by a semi-group approach using results for L^p-initial data derived by Kashiwabara and Hieber.

Compressible fluid model of Korteweg type with free boundary condition: model problem

Hirokazu Saito

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In this talk, we would like to consider a resolvent problem on the upper half-space $\mathbf{R}^N_+ = \{(x', x_N) \mid x' = (x_1, \ldots, x_{N-1}) \in \mathbf{R}^{N-1}, x_N > 0\}$ $(N \geq 2)$ arising from a compressible fluid model of Korteweg type with free boundary condition.

It is proved that there exist $\mathscr{R}$-bounded solution operator families of the resolvent problem in the following way: We first apply the partial Fourier transform with respect to the tangential variables x' to the resolvent problem in order to obtain ordinary differential equations with respect to x_N in the Fourier space. Secondly, we solve the ordinary differential equations. Thirdly, applying the inverse transform to the solution of ODEs gives the representation formula of solutions to the resolvent problem. Finally, we analyze the symbols (especially, Lopatinskii determinant) of the representation formula in detail in order to prove the existence of $\mathscr{R}$-bounded solution operator families mentioned above.

The $\mathscr{R}$-boundedness obtained above yields the maximal regularity for a time-dependent linear problem associated with the resolvent problem.

Stability estimates for continuity equations

Christian Seis

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In their ground breaking '89 paper, R. J. DiPerna and P.-L. Lions study uniqueness and stability of solutions to linear continuity equations with Sobolev coefficients. What is not contained in their "theory of renormalized solutions" are quantitative stability estimates which allow to control the distance of two solutions if the data are varied. Such estimates, however, are indispensable in the analysis of a number of applications in fuid dynamics.

In this talk, I will present a new quantitative approach to continuity equations that is based on such stability estimates. I will show how these estimates can be applied to bound the order of convergence of the numerical upwind scheme or the rate of mixing by stirring of fluids. This is partially joint work with A. Schlichting.

Enhancement of flow measurements using fluid-dynamic constraints

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Novel experimental modalities acquire spatially resolved velocity measurements for steady state and transient flows which are of interest for engineering and biological applications. One of the drawbacks of such high resolution velocity data is their susceptibility to measurement errors. We propose a novel filtering strategy that allows enhancement of noisy measurements to obtain reconstruction of smooth divergence free velocity and corresponding pressure fields, which together approximately comply to a prescribed flow model. The main step in our approach consists of the appropriate use of the velocity measurements in the design of a linearized flow model which can be shown to be well-posed and consistent with the true velocity and pressure fields up to measurement and modeling errors. The reconstruction procedure is formulated as a linear quadratic optimal control problem and the resulting filter has analyzable smoothing and approximation properties. We also discuss briefly the discretization of our approach by finite element methods and comment on the efficient solution of the linear optimality system by iterative solvers. The capability of the proposed method to significantly reduce data noise is demonstrated by numerical tests in which we also compare to other methods like smoothing and solenoidal filtering.

This is joint work with H. Egger and C. Tropea (TU Darmstadt).

Time-Periodic Non-Autonomous Evolution Equations in Interpolation Spaces

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We consider time-periodic solutions to abstract non-autonomous differential equations. Under the assumption of boundedness and a certain polynomial decay of the associated evolution family, we will be able to ensure the existence and stability of such solutions in a suitable interpolation space. The abstract theory can be applied to non-autonomous parabolic equations and non-autonomous Ornstein-Uhlenbeck equations with time-periodic coefficients as well as Navier-Stokes equations in unbounded domains. This work is a generalization of results by Yamazaki (2000) and Geissert, Hieber, Nguyen (2016).

The existence of $\mathscr{R}$-bounded Solution Operators of The Thermoelastic Plate Equation With Dirichlet Boundary Conditions

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In this talk, I will discuss the existence of $\mathscr{R}$-bounded solution operators of thermoelastic plate equation with the Dirichlet boundary condition case in general domain (Ω). The problem is formulated by the equation systems:

$$u_{tt} + \Delta^2 u_t + \Delta\theta = f_1 \text{ and } \theta_t - \Delta\theta - \Delta u_t = f_2 \text{ in } (0,\infty)\times\Omega$$

with initial and boundary conditions $u|_{t=0} = u_0, D_t u|_{t=0} = v_0, \theta|_{t=0} = \theta_0$ and $u|_{t=0} = D_n u|_{x_n=0} = \theta|_{x_n=0} = 0$ respectively, where $u = u(x,t)$ denotes a vertical displacement at time t at the point $x = (x_1, \cdots, x_n) \in \Omega$ while $\theta = \theta(x,t)$ describes the temperature.

In 2009, Naito and Shibata studied the analytic semigroup of the problem in the half-space with Dirichlet boundary condition and then, in 2016, Denk and Shibata proved the maximal $Lp - Lq$-regularity for the problem with free boundary condition in general domain. In this work, I prove the existence of the $\mathscr{R}$-bounded solution operator by using the idea due to Denk and Shibata in the free boundary condition case.

Global solutions for the incompressible rotating stably stratified fluids

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In this talk, we consider the initial value problem of the 3D incompressible Boussinesq equations for rotating stably stratified fluids. We establish the dispersive and the Strichartz estimates for the linear propagator associated with both the rotation and the stable stratification:

$$e^{itp_\mu(D)}f(x) := \int_{\mathbb{R}^3} e^{ix\cdot\xi \pm itp_\mu(\xi)}\widehat{f}(\xi)\,d\xi, \qquad p_\mu(\xi) := \frac{\sqrt{\xi_1^2+\xi_2^2+\mu^2\xi_3^2}}{|\xi|}.$$

Here, the real constant $\mu \in \mathbb{R}\setminus\{0,\pm1\}$ represents the ratio of the angular frequency of rotation and the buoyancy frequency. As an application, we show a unique existence of global in time solutions to our system for some large class of initial data when the rotation speed and the buoyancy frequency are sufficiently high.

This talk is based on the joint work with Alex Mahalov (Arizona State University) and Tsukasa Iwabuchi (Tohoku University).

Recent results on the Stokes and Navier-Stokes equations on bounded Lipschitz domains

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In this talk, I will give an overview of some of the recent results dealing with the Stokes and Navier-Stokes equations on a bounded, three-dimensional Lipschitz domain. More precisely, to perform certain iteration or fixed point methods to prove the existence of solutions to the Navier-Stokes equations in L^p-spaces, several properties of the Stokes operator and the Stokes semigroup are needed. All of the required properties will be presented as well as the corresponding existence results to the Navier-Stokes equations.

Extending a hybrid Level Set / Front Tracking method for the simulation of surface tension driven flows.

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When performing numerical simulations of multiphase flows the results are impaired by so-called *spurious currents*. While this impact may be acceptable for flows dominated by convective forces, it can prohibit the simulation of flows driven by capillary forces. New insights in the cause of spurious currents and in ways to reduce them have been gained in recent years. Still, the reduction of spurious currents remains an active research topic for simulations of multiphase flows using unstructured meshes.
In this talk some of the difficulties concerning spurious currents in the context a hybrid Level Set / Front Tracking method are discussed. They are illustrated with the help of a stationary droplet. Among the discussed points are the discretized Navier Stokes equations, the modelling of surface tension and of required geometric quantities.
This talk is based on joint work with Tomislav Maric and Prof. Dieter Bothe from the *Mathematical Modelling and Analysis* group, TU Darmstadt.

Critical spaces for the Navier-Stokes equations with Navier boundary conditions

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We prove that the n-dimensional Navier-Stokes equations subject to Navier boundary conditions generate a local semiflow in the critical spaces $B_{qp}^{n/q-1}$. The proof is based on the theory of weighted L_p-maximal regularity for abstract semilinear evolution equations and theextrapolation-interpolation scales.

Regularity structures for the primitive equations

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The new technique of regularity structures introduced by Martin Hairer is a method to find local solutions for subcritical SPDEs with rough data, which were formerly inaccessible. Regularity structures are spaces of abstract "Taylor expansions", in which a solution of a SPDE can be found by solving fixed point equations. The sequence of these fixed points, which are renormalised solution of the SPDE, converges to a solution of the SPDE. We will give a short introduction how to modify them to solve the primitive equations with white noise.

Two-phase model of crowd propagation

Ewelina Zatorska

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I will talk about the fluid equations used to model pedestrian motion and traffic. I will present the compressible-incompressible Navier-Stokes two phase system describing the flow in the free and in the congested regimes, respectively. I will also show how to approximate such system by the compressible Navier-Stokes equations with singular pressure for the fixed barrier densities and also some recent developments for the barrier densities varying in the space and time. This is a talk based on several papers in collaboration with: D. Bresch, C. Perrin, P. Degond, and P. Minkowski.

3 Participants

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