September 12, 2020

As was recently pointed out in [1], there is an oversight in the proof of Theorem 2.7 and, in fact, part (ii) of this theorem (while correct for formulas not containing v) needs to have a restricted double-negation-shift principle $U_k\text{-DNS}$ added to the verifying theory which e.g. follows from $\neg\neg(\Pi^0_k \lor \Pi^0_k)\text{-DNE}$ (see [2] for a detailed study of such principles). This has no consequences for the rest of the paper (in particular Corollaries 2.8, 2.9 remain valid) but leaves open the possibility that $E_k\text{-DNE}$ might be strictly stronger than $\Sigma^0_k\text{-DNE}$. In any case, $E_k\text{-DNE}$ has the same relation to all other principles in Figure 2 as $\Sigma^0_k\text{-DNE}$ has since it still follows from $\Sigma^0_k\text{-LEM}$ (which is equivalent to $E_k\text{-LEM}$ by Corollary 2.9) while it does not imply $\Sigma^0_k\text{-LLPO}$ (Proof for $k = 1$: $E_1\text{-DNE} \subseteq \Sigma_1\text{-DNE} + U_1\text{-DNS}$ has a direct (without negative translation) functional interpretation by bar-recursive functionals while $\Sigma^0_1\text{-LLPO}$ has not (since it has $\Pi^0_3$-consequences without computable Skolem functions).