Ob dies die Beh. I, 1.28(i) im Compendium widerlegt? | Erné |

An example of an algebraic (hence continuous) poset containing an interval which is not even $\wedge$-continuous.

$C := \{1 - \frac{1}{n} : n \in \mathbb{N}\} \cup \{1\}$,

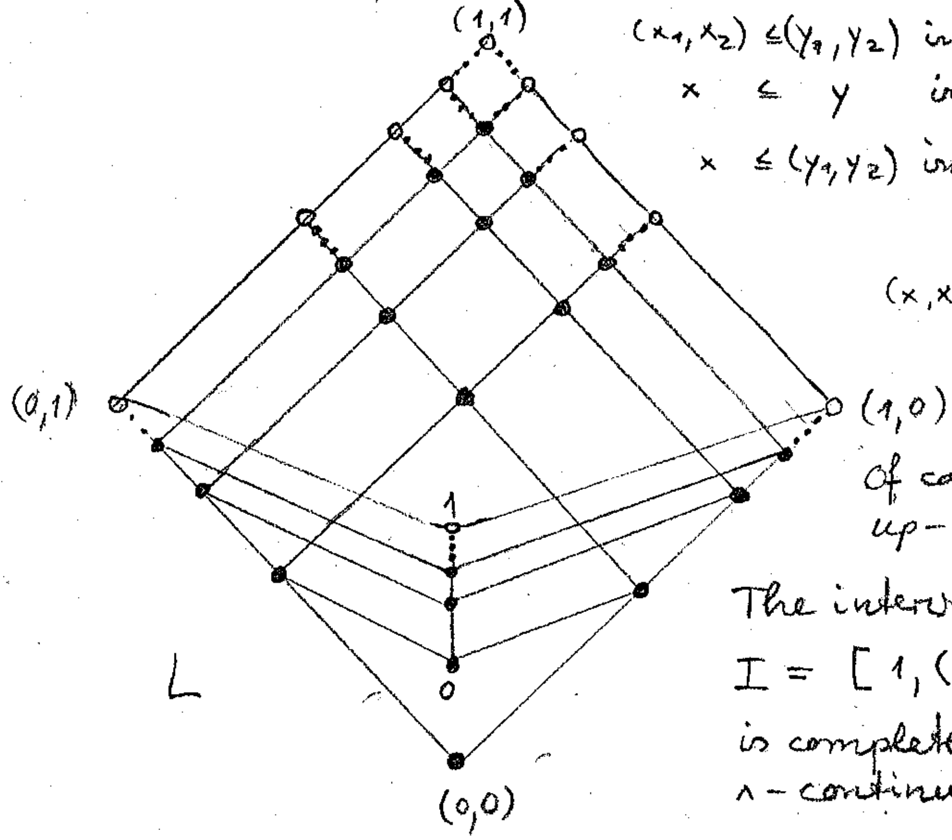
$L := (C \times C) \cup C$, partially ordered by

$(x_1, x_2) \le (y_1, y_2)$ in $L \iff x_1 \le y_1,\ x_2 \le y_2$

$x \le y$ in $L \iff x \le y$ in C

$x \le (y_1, y_2)$ in $L \iff x < y_1$ or $x < y_2$ or $x = y_1 = 1$ or $x = y_2 = 1$

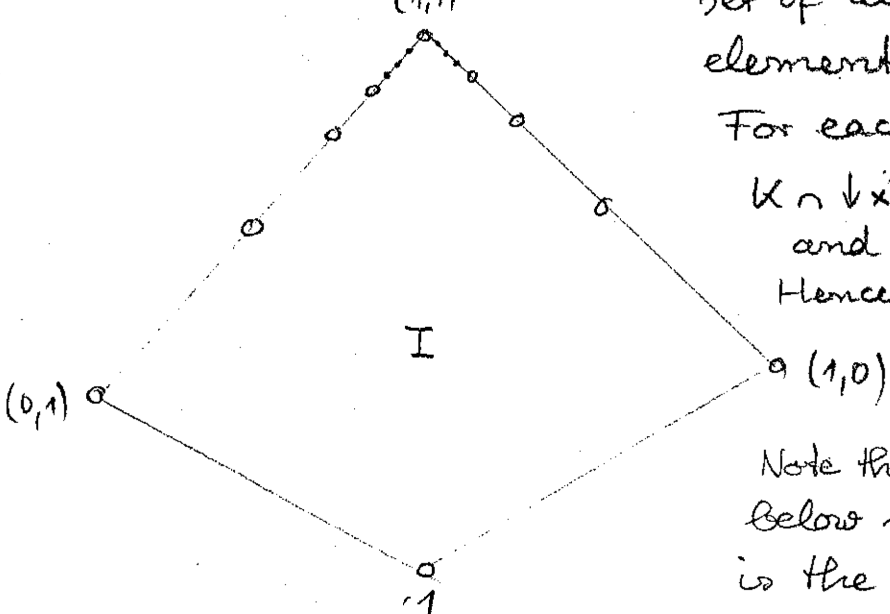
$(x, x_1, x_2, y, y_1, y_2 \in C)$



Of course, L is up-complete.

The interval $I = [1, (1,1)]$ is complete but not $\wedge$-continuous.

$K = L \setminus I$ is the set of all compact elements of L. For each $x \in L$, $K \cap \downarrow x$ is directed, and $\bigvee(K \cap \downarrow x) = x$. Hence L is algebraic.

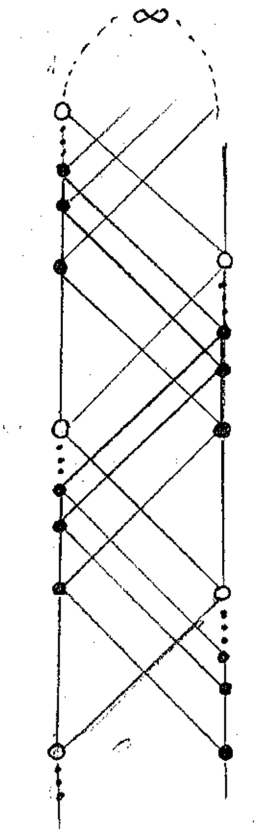


Note that each way-below set in I is the singleton $\{1\}$, so that I is "extremely non-continuous".

Continuous Poset with an interval

Another example:

Let $C = \{ m - \frac{1}{n} : m \in \mathbb{Z}, n \in \mathbb{N} \}$.



black: compact elements

Define a partial order on $C \times \{0,1\}$ as follows

$$(x,a) \le (y,b) \iff \begin{array}{l} x \le y \text{ and } a = b \\ \text{or } x \le y - \frac{1}{2},\ a = 0,\ b = 1 \\ \text{or } x \le y - \frac{1}{2},\ a = 1,\ b = 0, \end{array}$$

and adjoin a greatest element ∞.

Then $(C \times \{0,1\}) \cup \{\infty\}$ is an algebraic poset, but none of the intervals

$[(x,0), (x+1,0)] \quad (x \in \mathbb{Z})$

is even $\wedge$-continuous.