/KEIMEL/

Dear Karl,

Here are some observations on the problem we were considering in San Antonio.

Let K be a compact convex subset of a topological vector space. Let $C(K)$ be the space of all compact convex subsets of K; note this is a closed subset of all closed subsets of K and hence is compact.

Under the partial order of inclusion $C(K)$ becomes a semilattice where

$$K_1 \wedge K_2 = \text{closed convex hull}(K_1 \cup K_2)$$
$$= \{ta + (1-t)b : a \in K_1,\ b \in K_2,\ 0 \le t \le 1\}.$$

The last line can be used to show $C(K)$ is a topological semilattice. If K sits in a locally convex space, then $C(K)$ is a $\mathcal{CL}$ object. (It follows from my work and Roberts' that if $C(K)$ is a $\mathcal{CL}$ object, then ~~[illegible]~~ K can be affinely embedded in a locally convex space.)

From here on we assume $C(K)$ to be a $\mathcal{CL}$ object, i.e., we assume K sits in a locally convex space. Note that $p \in K$ implies that $\{p\}$ is a prime in $C(K)$ if and only if p is an extreme point of K.

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Now there exist examples of compact convex sets in which the extreme points are dense. Since the closure will contain all singleton sets in $C(K)$, it follows that the primes form a generating set. However $C(K)$ is not a distributive lattice.

It is interesting to note in this context that Krein–Milman says something about the existence of primes. Is Krein–Milman really a $\mathcal{CL}$-theorem?

Regards,