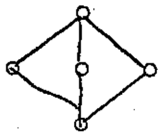
Keimel

COmmentary on the preprint by Gierz and Keimel, Corrigenda.

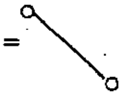
Quiquid id est timeo Danaos et dona ferentes

Vergil

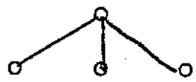
GK point out that my proof of 3.17 on p.20 is erroneous ~~xxxxxxxxx~~.
They insist, that one has to use ~~xxxx~~ more decisively that all λ are lattice congruences. Their illustration is the semilattice T =



$\in \underline{Z}_L$.

Suppose we take $T[\lambda_1]$ =  , $T=[\lambda_2]$ =  , $T[\lambda_3]$= 

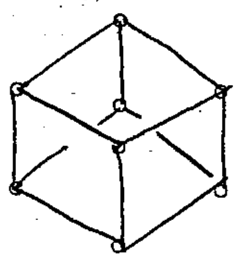
hence $X = \cup_k T[\lambda_k]$ =



$\in \underline{ZP}$

The functor G = $\hat{\hat{}}$ on $\underline{ZP}$ reads this partial semilattice as the upper cap of the cube and makes the cube from it:

GX =



$\in \underline{Z}$

and the morphism GX——>T will collapse the bottom ideal of the cube which is in contradiction to my proof.

I would also like to take this opportunity to rectify a mix-up in terminology and notation in my earlier notes. In 1.1 -1.3 on p. ~~2~~ 1,2 I talk about left adjoints of $\underline{Z}$ -morphisms when I should have talked abou right adjoints. The maps g should have better been called d in according to ~~XXXXXX~~ ATLAS. (Everything said about their properties standa, but read "right" when I say "left".) ~~Thisxxxxxxxxxxxxxxx~~ ~~the proof of 3.16xxxxxxxxxx~~ Please designate the REMARK on p.19 as REMARK 3.16_a and replace the entire old page 20 by the attached page 20.

the SP -situation. However, GK have the following clever situation (which supersedes 2.11): [Note the sly use of all hypotheses]

3.16$_b$. PROPOSITION. Let $T \in \mathcal{Z}_L$ and let $X = \bar{X} \subseteq T$ be a union of a collection $T[\lambda]$, $\lambda \in \Lambda$, of subobjects in $\mathrm{Sub}_L(T)$. Let $\varphi: X \longrightarrow 2$ be a continuous function. Then the following statements are equivalent

(1) φ is a character of $(X, D_X, .)$.

(2) $\varphi|T[\lambda]$ is a character for all $\lambda \in \Lambda$, and φ is monotone

(3) There is a character $\phi: T \longrightarrow 2$ such that $\varphi = \phi|X$.

Proof. (3)=>(1)=>(2) is trivial. We prove (2)=>(3): We define $F = \varphi^{-1}(1)$. Since φ is continuous, F is open closed in X; since φ is monotone, $\uparrow F = F$. Let $k \in K(T)$. Then $\uparrow k \cap X \subseteq F$ iff $\bigcup_\Lambda(\uparrow k \cap T[\lambda]) = \uparrow k \cap \bigcup_\Lambda T[\lambda] \subseteq F$ iff $\uparrow k \cap T[\lambda] \subseteq F$ for all $\lambda \in \Lambda$. If $d_\lambda: T \longrightarrow T[\lambda]$ is the right adjoint of the inclusion then $d_\lambda(k) = \inf \uparrow k \cap T[\lambda]$. Since $\varphi|T[\lambda]$ is a character, by (2), then $F \cap T[\lambda] = (\varphi|T[\lambda])^{-1}(1)$ is a ~~n open closed~~ filter of $T[\lambda]$. Thus $\uparrow k \cap T[\lambda] \subseteq F$ iff $d_\lambda(k) \in F$. Now we let $A = \{k \in K(T): d_\lambda(k) \in F \text{ for all } \lambda \in \Lambda\} = \bigcap_\Lambda d_\lambda^{-1}(F) \cap K(T)$. Then clearly $\uparrow_{K(T)} A = A$. We claim

(1) A is an inf subsemilattice of K(T).

Indeed let $k, h \in A$. Then $d_\lambda(kh) = d_\lambda(k)\, d_\lambda(h)$ since d_λ is a lattice morphism by 1.1 and 1.4 (recall notational correction!) and the hypothesis that $T[\lambda] \in \mathrm{Sub}_L(T)$. Since $F \cap T[\lambda]$ is a filter of $T[\lambda]$ and $d_\lambda(k), d_\lambda(h) \in F \cap T[\lambda]$ (because of $k, h \in A$) then $d_\lambda(kh) \in F$, showing $kh \in A$, and proving claim (1). Claim

(2) $F \subseteq \uparrow A$.

Indeed if $x \in F$, then, since F is open in X there is an open neighborhood U of x in T with $U \cap X \subseteq F$. Since $T \in \mathcal{Z}$, we may assume that U is also closed and that $k = \min U$ exists. Then $\uparrow k \cap X \subseteq \uparrow U \cap X \subseteq \uparrow F \cap X = F \cap X$, showing $k \in A$. Since $k \leq x$, (2) is proved. Now (2) shows that $F \subseteq \bigcup \{\uparrow k: k \in A\}$; since F is compact there is a finite set $E \subseteq A$ with $F \subseteq \bigcup \{\uparrow k: k \in E\} = \uparrow \bigwedge E$. But $k_o = \bigwedge E \in A$ by (1). Now $F \subseteq \uparrow k_o \cap X \subseteq F$. Let ϕ be the character with $\phi^{-1}(1) = \uparrow k_o$. □

3.17. COROLLARY. Let $T \in \mathcal{Z}_L$, $X = \bar{X} = \bigcup\{T[\lambda]: \lambda \in \Lambda\}$ for some separating collection $\Lambda \subseteq \mathrm{Cong}_L(S)$ (S = dual of T). Then $GX \longrightarrow T$ is an isomorphism.

Proof. By 2.5, X is generating, so $GX \longrightarrow T$ is surjective 3.15. By 3.16$_a$ and 3.16$_b$, $GX \to T$ is injective in view of 1.6. □