SEMINAR ON CONTINUITY IN SEMILATTICES (SCS) 17a

NAME(S):	GIERZ KEIMEL	DATE M	D	Y
		october 5	76	

TOPIC An Error in the copower considerations

REFERENCE HOFAMNN SCS-Memoir from 9/2o/76

In the SCS-Memoir cited above, the following assertion is used in the proof of lemma 2.4:

For every compact space S , $\beta(J\times S) \cong \beta J \times S$.

This is wrong for two reasons: Firstly, in Trans. Amer.Math. Soc. 90 (1959), p.369 ff., Glicksberg has shown that $X\times Y$ is pseudocompact if and only if $\beta(X\times Y) \cong \beta X \times \beta Y$ (A space is pseudocompact if every ~~bounded~~ realvalued continuous function on it is bouded), provided that X and Y are both infinite.

More concretely, let us consider the example, where $J = \mathbb{N}$ and $S = I$ the unit interval.

Let $x_n = (1/n,n)$ and $y_n = (0,n)$ for all natural numbers n .

For every free ultrafilter u on $\mathbb{N}$, y_n and x_n converge to the same point $(0,u)$ in $\beta\mathbb{N}\times I$. At the other hand, let $f\colon \beta\mathbb{N}\times I \longrightarrow I$ be defined by $f(n,a) = \min(na,1)$. Then f is continuous on $\mathbb{N}\times I$, and $f(x_n) = 1$, $f(y_n) = 0$ for all n .

Thus, f cannot be extended continuously onto all of $\beta\mathbb{N}\times I$.

This also shows that $\beta(\mathbb{N}\times I) \not\cong \beta\mathbb{N}\times I$.

Note, that f is a <u>CL</u>-morphism on all the fibers $\{n\}\times I$. Thus, this example also shows that $\beta\mathbb{N}\times I$ does not generate the copower $^{\mathbb{N}}I$.

The coproduct of <u>CL</u>-objects L_i can abstractly be constructed in the following way: Let L be the topological sum of the L_i . Let F be the set of all continuous maps from L into I which are <u>CL</u>-morphisms on each L_i . Then L is canonically embedded in I^F under evaluation. The <u>CL</u>-subobject of I^F generated by the image of L is the coproduct of the L_i . The closure of the image of L in I^F is, what we would like to know.

West Germany: TH Darmstadt (Gierz, Keimel)
U. Tübingen (Mislove, Visit.)

England: U. Oxford (Scott)

USA: U. California, Riverside (Stralka)
LSU Baton Rouge (Lawson)
Tulane U., New Orleans (Hofmann, Mislove)
U. Tennessee, Knoxville (Carruth, Crawley)

s. Rückseite

L_u

This closure of the image of L is a union of continuous lattices which are disjoint except for the identity; one continuous lattice for each ultrafilter on J. The breadth of these lattices does not exceed the maximum breadth of the L_i, if this maximum is finite. In particular, if all the L_i are totally ordered, the same holds for the L_u which come into play. The coproduct of the L_i is isomorphic to the lattice of all closed subsets of the union of the L_u, $u \in \beta J$, which meet every L_u in a filter. These filters may be viewed as lower semicontinuous functions.

We mentally proved these assertions, but we did not write them down because we do not know whether it is worthwile. So the correctness cannot be guaranteed.