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TOPIC

The Fell compactification

REFERENCE

Rudolf-E. Hoffmann, The Fell compactification revisited, manuscript

Anyone interested in a copy of the manuscript may write to me.

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ABSTRACT:

The Fell compactification $\underline{H}(X)$ of a locally quasi-compact T_0 -space X can be viewed as a compact ordered space. Then $\underline{H}(X)$ corresponds to a quasi-compact, locally quasi-compact super-sober space ψX whose open sets are all the open upper sets of $\underline{H}(X)$. There is an essential extension $X \hookrightarrow \psi X$ in the category \underline{T}_0 of T_0 -spaces and continuous maps.

We show that

$$\underline{O}(\psi X) \cong \text{DID}(L)$$

for the distributive continuous lattice $L = \underline{O}(X)$ - where $\underline{O}(Y)$ is the lattice of open sets of a space Y , $D(P)$ is the dual of a continuous poset P , and $I(P)$ is the continuous lattice underlying the injective hull of P (endowed with the Scott topology σ_P) in the category \underline{T}_0 .

This result relies upon a representation of $\text{ID}(L)$ for a continuous $1, \wedge$ -semilattice L , viz.

$$\text{ID}(L) \cong \text{Filt}_c L,$$

the (continuous) lattice of all those filters of L which are generated by Scott-open subsets of L . For a distributive continuous lattice L , the meet-prime elements of $\text{DFilt}_c L$ in their (hull-kernel) topology are (topologically) identified with the pseudo-meet-prime (=weakly meet-prime) elements of L endowed with the \sqcap -topology of L^{op} .

Furthermore both $\underline{H}(?)$ and $\psi(?)$ are shown to be functorial on the category of locally quasicompact T_0 -spaces and continuous perfect mappings.