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SEMINAR ON CONTINUITY OF SEMILATTICES

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		7	16	82

Topic: A remark on the complete distributivity of algebraic lattices

REFERENCES:

HMS Hofmann,K.H.,M.Mislove, and A.Stralka, The Pontryagin duality of compact 0-dimensional semilattices, LNM 396 (1974)

H-79 Hofmann,K.H., Completely distributive algebraic lattices, SCS-Memo 11-27-79.

H-83 _ , Complete distributivity and the essential hull of a T_0 space, Proc.Bremen Workshop Cont.Lattices 1982, see also SCS-Memo 6-8-82.

HM Hofmann,K.H. and M.Mislove, Free objects in the category of completely distributive lattices, ibid., see also SCS-Memo 11-24-81.

C A Compendium

We all know very well that various degrees of distributivity and various degrees of abundance of prime or completely prime elements are interrelated; in HMS-Duality we started on this theme on 53-78. In particular, we observed at a very early stage that on algebraic lattices, distributivity does not imply join continuity (loc.cit. p.56). In the Compendium, Chapter VII, Section 2, pp.316 ff. notably in 2.4,2.8,2.9 and 2.12 Jimmie Lawson elaborated on the delicate interplay in distributive continuous lattices of join continuity, complete distributivity, bicontinuity and the continuity of the finitary sup-operation. In particular, in loc.cit.2.12 he showed that

there is a continuous distributive lattice L *which is join continuous but which is not completely distributive.*

Notice that it is not easy to come by this example since it involves the construction of compact unital topological semilattices without small subsemilattices.

I do not know whether it has been noticed that it would be much harder to construct a join continuous ALGEBRAIC distributive lattice which is not completely distributive. In fact:

OBSERVATION. *Every join Brouwerien algebraic lattice is completely distributive.*

Remark. Recall that "join Brouwerien" = " distributive + join continuous".

Proof. In the proof of [C], p.92, Proposition 4.21 we showed that for each completely irreducible element $p \in \mathrm{Irr}\, L$ the set $L \setminus \downarrow p$ has a minimum k. Thus L is the disjoint union of $\downarrow p$ and $\uparrow k$. This means that p is completely prime (and k is completely coprime). Thus all elements of Irr L are completely prime, and since Irr L is order generating, this means that L is completely distributive.(Cf. HMS loc.cit,, H-79) □

Is this observation explicitly somewhere in the accessible literature? Of course, one can rephrase the observation by saying that *an algebraic lattice is completely distributive if and only if it is join Brouwerien.*