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TOPIC: A remark on congruence lattices of lattices

REFERENCES


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3. E. T. Schmidt, Zur Charakterisierung der Kongruenzverbände der Verbände, Mat.-Fyz. Časopis Slovensk. Akad. Vied 18 (1968), 3-20.
4. E. T. Schmidt, The ideal lattice of a distributive lattice with 0 is the congruence lattice of a lattice, Acta Sci. Math. 43 (1981), 153-168.


Congruence lattices of lattices are distributive and algebraic. It is one of the longest-standing conjectures in lattice theory that the converse is also true:

CONJECTURE. Every distributive algebraic lattice L is isomorphic to the congruence lattice of some lattice.

In 1968, E. T. Schmidt [3] reduced this conjecture as follows:

THEOREM A. Suppose that there exist a generalized Boolean lattice B and a distributive congruence $\Theta$ on B such that $L^c \cong B/\Theta$, where $L^c$ denotes the distributive sup-semilattice with zero consisting of all compact elements of L. Then L can be represented as the congruence lattice of some lattice.



Later, in 1981 Schmidt [4] has used this result to show that the above conjecture is true if $L^c$ is a lattice. On the other hand, Heiko Bauer (unpublished) has verified the conjecture for countable $L^c$. Subsequently we shall describe an easy way to obtain from Theorem A that L is representable if $L^c$ is locally countable (i. e., every principal ideal of $L^c$ is countable).

Motivated by investigations of isomorphism types of countable Boolean lattices, Dale Myers introduced certain measures on Boolean lattices with values in refinement monoids. In the papers [1, 2] we studied these measures, and as a particular consequence it is shown (see [2, p. 16]):

THEOREM B. If K is a locally countable distributive sup-semilattice with zero then there are a locally countable generalized Boolean lattice B and a V-congruence $\Theta$ on B such that $K \cong B/\Theta$.

It is trivial that each V-congruence is weak-distributive. Moreover, it is not difficult to see that each weak-distributive congruence on a locally countable generalized Boolean lattice is in fact distributive. Therefore one concludes from Theorem A and Theorem B:

THEOREM C. Every distributive algebraic lattice L with locally countable $L^c$ is representable as the congruence lattice of some lattice.