Assumptive hypersequent-based argumentation

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PhDs in Logic VIII

Research Group for
Non-Monotonic Logic
and Formal Argumentation

RUB

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Project

- Research is part of the project *An Argumentative Approach to Defeasible Reasoning: Towards a Unifying Base Theory*
- By the Research Group for Non-Monotonic Logic and Formal Argumentation
- At the Institute of Philosophy II, Ruhr-Universität Bochum

Outline

Aim of this talk

- Generalization of the sequent approach to argumentation to hypersequents
- Use this generalization to give an argumentative approach to defeasible reasoning with normality assumptions

Defeasible reasoning (DR)

- Truth of the conclusion not warranted by the truth of the premises
- Dynamic: inferences can be retracted in view of new information

Defeasible reasoning (DR)

- Truth of the conclusion not warranted by the truth of the premises
- Dynamic: inferences can be retracted in view of new information
- **Argumentation:** a conclusion is drawn unless/until it is attacked
- **Adaptive logics:** formulas are derived on explicit and defeasible normality assumptions. In an abnormal situation inferences are retracted

Argumentation theory

- One way of modeling DR is by argumentation frameworks (AFs)
- Abstract AFs were introduced by Dung
- These AFs are directed graphs where
  - Nodes are abstract representations of arguments
  - Arcs represent argumentative attacks

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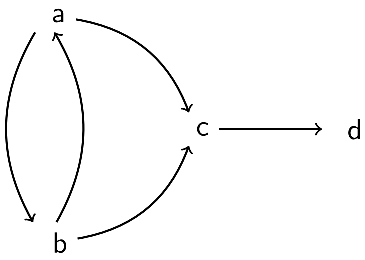
- $a \rightarrow b$ can be read as "a attacks b"
- Acceptance of arguments is calculated by argumentation semantics

Argumentation semantics

There are many argumentation semantics, here only the notions relevant for this talk:



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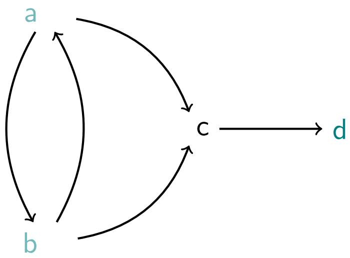

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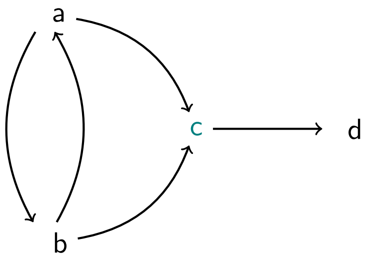

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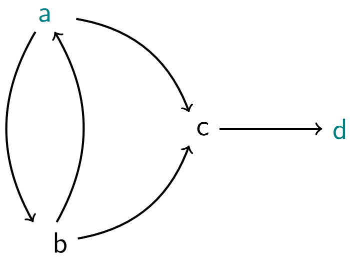

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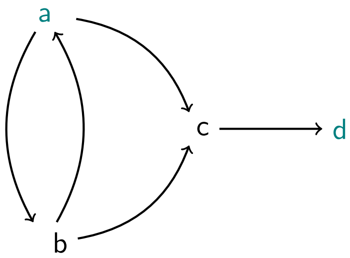

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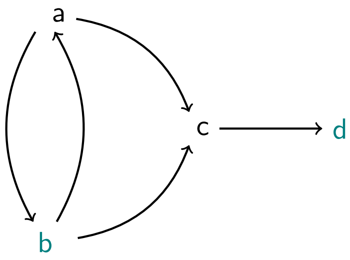

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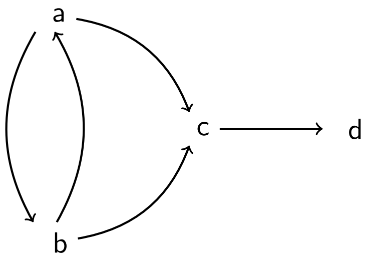

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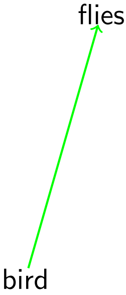

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- New information is obtained:
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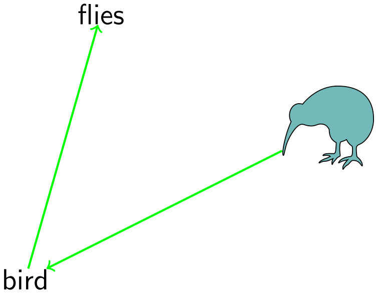

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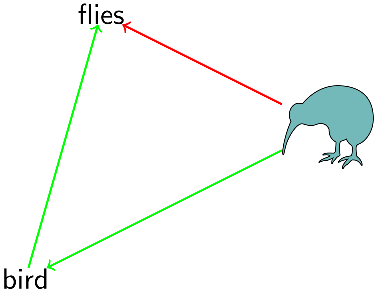
- Kiwis are **abnormal** birds: they do not fly!



Logical argumentation

- Dung's abstract AFs are sometimes not expressive enough
- Structured or logical argumentation
- Arguments are not abstract entities, but contain a logical structure
- Structure provided by formal languages
- Several forms, one of which is sequent-based argumentation

Sequent-based argumentation I

- Arguments are sequents provable in the core logic
- Constructing arguments is done by inference rules:

$$\frac{\Gamma_1 \Rightarrow \Delta_1 \quad \ldots \quad \Gamma_n \Rightarrow \Delta_n}{\Gamma \Rightarrow \Delta}$$

- Attacks are sequent elimination rules:

$$\frac{\Gamma_1 \Rightarrow \Delta_1 \quad \ldots \quad \Gamma_n \Rightarrow \Delta_n}{\Gamma_n \not\Rightarrow \Delta_n}$$

- Standard logical attacks have their own sequent elimination rule, e.g.:

$$\frac{\Gamma_1 \Rightarrow \psi_1 \quad \psi_1 \Rightarrow \neg \bigwedge \Gamma_2 \quad \Gamma_2 \Rightarrow \psi_2}{\Gamma_2 \not\Rightarrow \psi_2} \; \mathit{Def}$$

$$\frac{\Gamma_1 \Rightarrow \psi_1 \quad \psi_1 \Rightarrow \neg\psi_2 \quad \neg\psi_2 \Rightarrow \psi_1 \quad \Gamma_2 \Rightarrow \psi_2}{\Gamma_2 \not\Rightarrow \psi_2} \; \mathit{Reb}$$

Sequent-based argumentation II

- Acceptability is based on argumentation semantics applied to the resulting AF
- Advantages:
  - Different core logics, such as paraconsistent and deontic logics can be used
  - Arguments are automatically constructed and identified

Hypersequents

- Generalization of Gentzen's sequents:

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- Existence of finite cut-free hypersequent calculi for e.g. S5 and RM

Outline

Assumptive hypersequents

Idea: use hypersequents for the modeling of DR

- Reserve the leftmost component for normality assumptions

$$\Sigma \Rightarrow \Pi \mid \Gamma_1 \Rightarrow \Delta_1 \mid \ldots \mid \Gamma_n \Rightarrow \Delta_n$$

- External contraction, weakening and exchange rules cannot be applied to this component
- As long as the assumptions are false, one of the other components has to be true

Outline

Adaptive logics (ALs)

- Introduced by Batens in the early '80s
- Interpret the premises *as normally as possible*
- ALs offer a dynamic proof theory
- They explicate many forms of DR

Standard Format for ALs

Definition

In the standard format ALs consist of three components:

- Lower limit logic (LLL), a Tarski logic
- Set of abnormalities, depends on the application
- Adaptive strategy, defines which inferences are accepted:
  - Reliability, a more cautious form of reasoning
  - Minimal abnormality, a more credulous form

Semantics for the minimal abnormality strategy

- The abnormal part of a model M (denoted by $Ab(M)$) is the set of abnormalities validated by M
- The semantics selects all the models that validate a minimal set of abnormalities
- An LLL-model M of a premise set Γ is a *minimally abnormal model* of Γ iff for all LLL-models M' of Γ: $Ab(M') \not\subset Ab(M)$

Definition

Let $\mathcal{M}_{AL^m}(\Gamma)$ be the set of all minimally abnormal LLL-models of Γ. The *semantic consequence relation* is defined as:

$$\Gamma \Vdash_{AL^m} A \text{ iff for all } M \in \mathcal{M}_{AL^m}(\Gamma),\ M \vDash A$$

Some example ALs

- Paraconsistent logics
  - For example **CLuN** and **CLuNs**
  - Abnormalities of the form $\sim A \wedge A$
  - Interpretation of premises as consistent as possible
- Deontic logics
  - For example Goble's **P** and **DPM**
  - Abnormalities of the form $OA \wedge O\neg A$
  - Interpretation of premises as non-conflicting as possible

Hypersequent argumentation framework

Definition

$\mathsf{AF}_{\mathcal{L}}^{\Gamma} = \langle \mathcal{A}, \rightarrow \rangle$ an argumentation framework with premise set Γ and core logic $\mathcal{L}$ where

- $\mathcal{A} = \{\langle A, \Pi \rangle : \emptyset \Rightarrow \Pi \mid \Gamma' \Rightarrow A \text{ for some } \Gamma' \subseteq \Gamma\}$
- $\langle A, \Pi \rangle \rightarrow \langle B, \Theta \rangle$ where $A \in \Theta$

Hypersequent calculi for ALs

- There should be a (hyper)sequent calculus for the LLL
- One universal rule for adding abnormalities to the leftmost component
- For some abnormality $!A$:

$$\frac{\emptyset \Rightarrow \Pi \mid \Gamma \Rightarrow \Delta, !A \mid G}{\emptyset \Rightarrow \Pi, !A \mid \Gamma \Rightarrow \Delta \mid G} \; RC$$

- Acceptance of inferences depends on the strategy and is computed by means of Dung's semantics

Acceptance

Let $\mathsf{AF}^{\Gamma}_{LLL}$ be a hypersequent AF

Definition

A is *skeptically acceptable* in $\mathsf{AF}^{\Gamma}_{LLL}$ iff all preferred extensions of $\mathsf{AF}^{\Gamma}_{LLL}$ contain an argument with conclusion A

Definition

A is *freely acceptable* in $\mathsf{AF}^{\Gamma}_{LLL}$ iff there is an argument $a = \langle A, \Pi \rangle$ that is skeptically acceptable

Meta-theory

Let LLL be a lower limit logic with a cut-free (hyper)sequent calculus then:

Theorem

The hypersequent calculi for AL^r_{LLL} and AL^m_{LLL} admit cut-elimination

Theorem

$\Gamma \Vdash_{AL^m_{LLL}} A$ *iff* A *is skeptically acceptable in* AF_{LLL}

Theorem

$\Gamma \Vdash_{AL^r_{LLL}} A$ *iff* A *is freely acceptable in* AF_{LLL}

Lower limit logic CLuN

- A very weak paraconsistent logic
- Obtained by adding $A \vee \sim A$ to full positive classical logic
- A sequent system is obtained by dropping the negations rules of Gentzen's *LK* and adding:

$$\frac{\Gamma, A \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, \sim A} \Rightarrow \sim$$

- Abnormalities have the form $\sim A \wedge A$, in short $!A$
- *As normally as possible* means as few contradictions as possible are validated

Assumptive hypersequents for CLuN

- Every sequent $\Gamma \Rightarrow \Delta$ is changed into an assumptive hypersequent $\emptyset \Rightarrow \Pi \mid \Gamma \Rightarrow \Delta$

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Example proof

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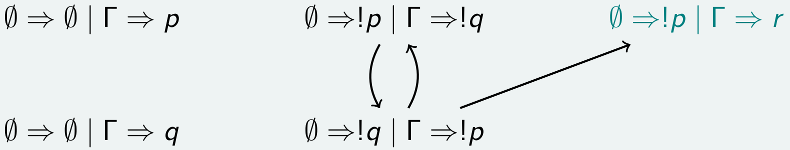

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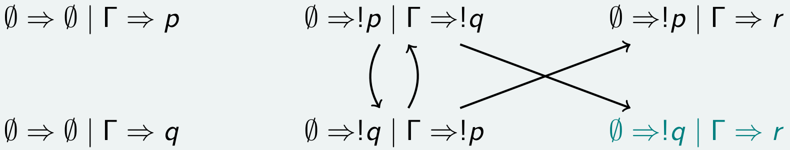

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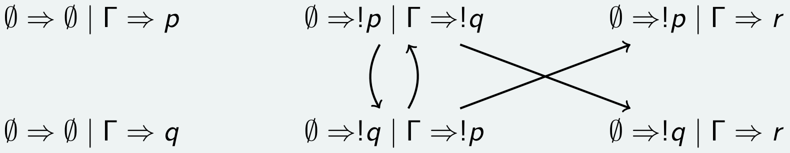

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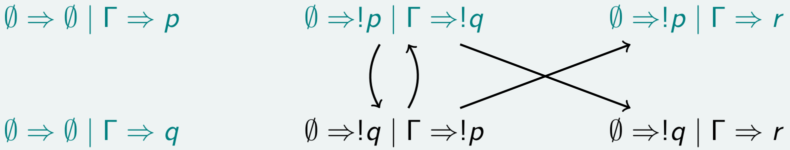

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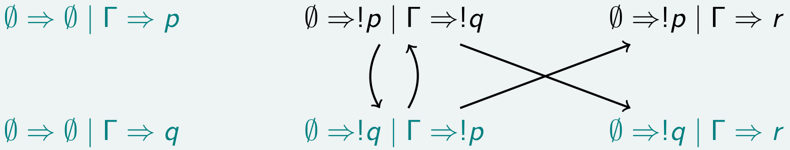

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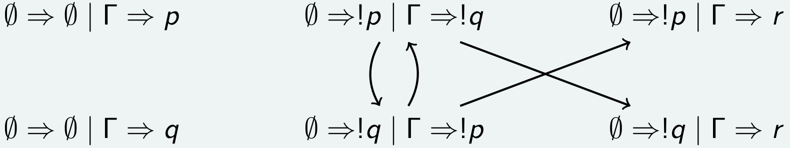

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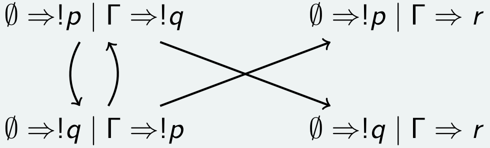

- r is skeptically acceptable: $\Gamma \vdash_{AL^m_{CLuN}} r$
- r is not freely acceptable: $\Gamma \nvdash_{AL^r_{CLuN}} r$

Outline

Conclusion

- The hypersequent system provides a highly expressive AF
- Any AL (with a (hyper)sequent calculus) can be embedded in it
- A supra-classical LLL is not needed
- Conflict management mechanisms of ALs are integrated within argumentation

Future research directions:

- Sequent rules for attacks
- Dynamic proof theory, useful for automated reasoning
- See how other uses of normality assumptions can be implemented
- The use of hypersequents to model priorities

Thank you!

Any questions or remarks?