Topological Modelling of Knowledge Change
Lecture 1

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PhDs in Logic VIII
Darmstadt, May 9th-10th, 2016

Outline

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Inductive Inference: Card Game

What is the rule behind this sequence of cards?

Inductive Inference: Card Game

What is the rule behind this sequence of cards?

A♠

Inductive Inference: Card Game

What is the rule behind this sequence of cards?

A♠ Q♠

Inductive Inference: Card Game

What is the rule behind this sequence of cards?

A♠ Q♠ 3♠

Inductive Inference: Card Game

What is the rule behind this sequence of cards?

A♠ Q♠ 3♠ A♠

Inductive Inference: Card Game

What is the rule behind this sequence of cards?

A♠ Q♠ 3♠ A♠ Q♠

Inductive Inference: Card Game

What is the rule behind this sequence of cards?

A♠ Q♠ 3♠ A♠ Q♠ 4♡

Inductive Inference: Card Game

What is the rule behind this sequence of cards?

A♠ Q♠ 3♠ A♠ Q♠ 4♡ . . .

How many different abstract scenarios?

Assume we have at our disposal unlimited amount of playing cards.

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1. How many different (kinds of) playing cards do we have?

How many different abstract scenarios?

Assume we have at our disposal unlimited amount of playing cards.

1. How many different (kinds of) playing cards do we have?
2. How many different beginnings of length 1?

How many different abstract scenarios?

Assume we have at our disposal unlimited amount of playing cards.

1. How many different (kinds of) playing cards do we have?
2. How many different beginnings of length 1?
3. How many different beginnings of length 2?

How many different abstract scenarios?

Assume we have at our disposal unlimited amount of playing cards.

1. How many different (kinds of) playing cards do we have?
2. How many different beginnings of length 1?
3. How many different beginnings of length 2?
4. How many different infinite sequences?

1.	A♠	A♠	A♠	A♠	A♠	A♠	...
2.	A♣	A♣	A♣	A♣	A♣	A♣	...
3.	A♡	A♡	A♡	A♡	A♡	A♡	...
4.	A♢	Q♠	3♠	8♡	2♡	5♠	...
5.	A♠	Q♠	7♠	J♠	5♠	5♠	...
...							
m.	A♣	A♡	A♣	A♡	A♣	A♢	...
...							

The Infinities of Inductive Inference

1.	A♠	A♠	A♠	A♠	A♠	A♠	. . .
2.	A♣	A♣	A♣	A♣	A♣	A♣	. . .
3.	A♡	A♡	A♡	A♡	A♡	A♡	. . .
4.	A♢	Q♠	3♠	8♡	2♡	5♠	. . .
5.	A♠	Q♠	7♠	J♠	5♠	5♠	. . .
. . .							
m.	A♣	A♡	A♣	A♡	A♣	A♢	. . .
. . .							

The Infinities of Inductive Inference

1.	A♠	A♠	A♠	A♠	A♠	A♠	...
2.	A♣	A♣	A♣	A♣	A♣	A♣	...
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...							
m.	A♣	A♡	A♣	A♡	A♣	A♢	...
...							

The Infinities of Inductive Inference

1.	A♠	A♠	A♠	A♠	A♠	A♠	. . .
2.	A♣	A♣	A♣	A♣	A♣	A♣	. . .
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. . .							

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...							

The Infinities of Inductive Inference

1.	A♠	A♠	A♠	A♠	A♠	A♠	...
2.	A♣	A♣	A♣	A♣	A♣	A♣	...
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...							
m.	A♣	A♡	A♣	A♡	A♣	A♢	...
...							

The Infinities of Inductive Inference

1.	A♠	A♠	A♠	A♠	A♠	A♠	...	
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...								
m.	A♣	A♡	A♣	A♡	A♣	A♢	...	m-th
...								

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. . .								
m.	A♣	A♡	A♣	A♡	A♣	A♢	. . .	m-th
. . .								
								. . .

How many possible rules are there?

1. In principle...

How many possible rules are there?

1. In principle...
2. Rule written down on a piece of paper.
3. Rule expressed by a natural language sentence.
4. Rule described by a theory that fills a 300 pages book.

How many possible rules are there?

1. In principle...
2. Rule written down on a piece of paper.
3. Rule expressed by a natural language sentence.
4. Rule described by a theory that fills a 300 pages book.
5. Rule encoded by a Turing Machine program.

Descriptions are finite, and there are countably many of them.

How many sequences comply to one rule?

1. The sequence has solely A♠-cards.

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3. The sequence has ♡-cards on even places.

How many sequences comply to one rule?

1. The sequence has solely A♠-cards.
2. The sequence has solely ♠-cards.
3. The sequence has ♡-cards on even places.
4. The sequence is definable in first-order logic.
5. etc...

Different Hypothesis Spaces

What can we know and how can we know it?

1. $\{(\text{all cards are } \spadesuit), (\text{all cards are } \diamondsuit)\}$
2. $\{(\spadesuit \text{ at the 4-th position}), \neg(\spadesuit \text{ at the 4-th position})\}$
3. $\underline{\{(\text{exactly } n \text{ cards are } \heartsuit) \mid n \in \mathbb{N}\}}$
4. $\{(\text{exactly } n \text{ cards are } \heartsuit) \mid n \in \mathbb{N}\} \cup \{(\infty \text{ cards are } \heartsuit)\}$

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A♠

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A♠ Q♠

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A♠ Q♠ 3♠ A♠ Q♠ 4♡ Q♠ Q♡...

The Game of Learning in the Limit: Learner and Nature

- A class of possible worlds (known by both players).
- Nature chooses one of them (learner does not know which).
- Nature generates data about the world.
- From inductively given data learner draws her conjectures.
- With each input learner can answer with a different hypothesis.
- Learner succeeds if

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- With each input learner can answer with a different hypothesis.
- Learner succeeds if **she stabilizes to a correct hypothesis**.

Her success depends on her skills and on the problem.

Outline

Function Learning

1 Possible realities:

Functions

2 Hypotheses:

Names of functions

3 Information accessible to the learner:

Sequences of pairs (argument, value)

4 Learner:

Function that takes a sequence and outputs a hypothesis

5 Success criterion:

After finite number of outputs stabilize on a correct answer

Learning Paradigms

Model-theoretic Learning

1. Possible realities:

   Models of a given signature

2. Hypotheses:

   First order sentences

3. Information accessible to the learner:

   Sequences of atomic formulas and negations thereof

4. Learner:

   Function that takes a sequence and outputs a hypothesis

5. Success criterion:

   After finite number of outputs stabilize on a correct answer

Set Learning

1. Possible realities:

   Sets of integers
2. Hypotheses:

   Names of sets
3. Information accessible to the learner:

   Sequences of numbers
4. Learner:

   Function that takes a sequence and outputs a hypothesis
5. Success criterion:

   After finite number of outputs stabilize on a correct answer

Outline

Another Game

Consider the following class:

$\{1, 2, 3, \quad 5, \ldots\}$
$\{1, \quad 3, 4, 5, \ldots\}$
$\{1, 2, \quad 4, 5, \ldots\}$
$\{\quad 2, 3, 4, 5, \ldots\}$
$\ldots$

Another Game

Consider the following class:
$\{1, 2, 3, \quad 5, \ldots\}$
$\{1, \quad 3, 4, 5, \ldots\}$
$\{1, 2, \quad 4, 5, \ldots\}$
$\{\quad 2, 3, 4, 5, \ldots\}$
$\ldots$

1

Another Game

Consider the following class:
$\{1, 2, 3, \quad 5, \ldots\}$
$\{1, \quad 3, 4, 5, \ldots\}$
$\{1, 2, \quad 4, 5, \ldots\}$
$\{ \quad 2, 3, 4, 5, \ldots\}$
...

1, 3

Another Game

Consider the following class:
$\{1, 2, 3, \quad 5, \ldots\}$
$\{1, \quad 3, 4, 5, \ldots\}$
$\{1, 2, \quad 4, 5, \ldots\}$
$\{\quad 2, 3, 4, 5, \ldots\}$
...

1, 3, 4

Another Game

Consider the following class:
$\{1, 2, 3, \quad 5, \ldots\}$
$\{1, \quad 3, 4, 5, \ldots\}$
$\{1, 2, \quad 4, 5, \ldots\}$
$\{\quad 2, 3, 4, 5, \ldots\}$
...

1, 3, 4, 2

Another Game

Consider the following class:

$\{1, 2, 3, \quad 5, \ldots\}$
$\{1, \quad 3, 4, 5, \ldots\}$
$\{1, 2, \quad 4, 5, \ldots\}$
$\{ \quad 2, 3, 4, 5, \ldots\}$
$\ldots$

1, 3, 4, 2, 6

Another Game

Consider the following class:

$\{1, 2, 3, \quad 5, \ldots\}$
$\{1, \quad 3, 4, 5, \ldots\}$
$\{1, 2, \quad 4, 5, \ldots\}$
$\{ \quad 2, 3, 4, 5, \ldots\}$
...

1, 3, 4, 2, 6, 7

Consider the following class:

$\{1, 2, 3, \quad 5, \ldots\}$
$\{1, \quad 3, 4, 5, \ldots\}$
$\{1, 2, \quad 4, 5, \ldots\}$
$\{ \quad 2, 3, 4, 5, \ldots\}$
...

1, 3, 4, 2, 6, 7, 8

Another Game

Consider the following class:

$\{1, 2, 3, \quad 5, \ldots\}$
$\{1, \quad 3, 4, 5, \ldots\}$
$\{1, 2, \quad 4, 5, \ldots\}$
$\{ \quad 2, 3, 4, 5, \ldots\}$
...

1, 3, 4, 2, 6, 7, 8, . . .

Another Game

1. Are you confident? What would make you change your guess?
2. What was your "guessing rule"?
3. How do you like winning if at least one of your guess is correct?
4. And if you succeed to make a right guess and never change your mind after that? How many wrong guesses could you make under this condition?

Another Game

1. Assume that I'll give you all and only truthful clues. What would be the guessing rule to win according to the last winning condition?
2. Add $\{1, 2, 3, 4, 5, \ldots\}$. Is your guessing rule still good?
3. While keeping $\{1, 2, 3, 4, 5, \ldots\}$ in, assume that I'll guarantee they are ordered increasingly. Can you win the game?
4. Now, remove $\{1, 2, 3, 4, 5, \ldots\}$. You get only one guess—would you object to this winning condition?

1960s: The Beginnings

- Hillary Putnam (1965). Trial and error predicates and the solution to...
- E. Mark Gold (1967). Language identification in the limit.
- Ray Solomonoff (1964). A formal theory of inductive inference.

Trial and Error Predicates

A predicate (set) P is decidable if there is a effective procedure φ such that

$$\begin{array}{lll} P(x) & \text{iff} & \varphi(x) = 1; \\ \neg P(x) & \text{iff} & \varphi(x) = 0. \end{array}$$

What happens if we modify the condition by:

1. allowing φ to change her mind any finite number of times;
2. making it impossible to diagnose termination?

P is a trial and error predicate if there is a Turing Machine φ such that

$$\begin{array}{lll} P(x) & \text{iff} & \exists k \forall n \geq k \; \varphi(x, n) = 1; \\ \neg P(x) & \text{iff} & \exists k \forall n \geq k \; \varphi(x, n) = 0. \end{array}$$

Trial and error predicates are decidable in the limit.

Trial and Error Predicates

A predicate (set) P is decidable if there is a recursive function φ such that

$$\begin{array}{lll} P(x) & \text{iff} & \varphi(x) = 1; \\ \neg P(x) & \text{iff} & \varphi(x) = 0. \end{array}$$

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Trial and error predicates are decidable in the limit.

Kleene-Mostowski Arithmetical Hierarchy

In this context one can think of φ as of a learning function,
Especially if more than two answers are possible.

The quantifier prefix in the definition of trial and error predicates indicates their place in arithmetic hierarchy.

$$
\begin{array}{ccccccccccccc}
 & & \boldsymbol{\Sigma}^0_1 & & & & \boldsymbol{\Sigma}^0_2 & \cdots & & & \boldsymbol{\Sigma}^0_\alpha & & & \cdots \\
 & \nearrow & & \searrow & & \nearrow & & & & \nearrow & & \searrow & & \\
\boldsymbol{\Delta}^0_1 & & & & \boldsymbol{\Delta}^0_2 & & & \cdots & \boldsymbol{\Delta}^0_\alpha & & & & \boldsymbol{\Delta}^0_{\alpha+1} & \cdots \\
 & \searrow & & \nearrow & & \searrow & & & & \searrow & & \nearrow & & \\
 & & \boldsymbol{\Pi}^0_1 & & & & \boldsymbol{\Pi}^0_2 & \cdots & & & \boldsymbol{\Pi}^0_\alpha & & & \cdots
\end{array}
$$

We will focus on a more general case,
when learner has to pick from more than two options,
in fact, from countably many options.

Some Basic Definitions

Let $\mathbb{N}$ stand for positive integers and $S \subseteq \mathbb{N}$.
Let $\mathcal{S} = (S_i)_{i \in \mathbb{N}}$ be a family of sets.

Definition

By a *stream* t for S we mean an infinite sequence of elements from S enumerating all and only the elements from S (allowing repetitions).

Definition

We will use the following notation:

- t_n is the n-th element of t;
- $t[n]$ is the sequence $(t_0, t_2, \ldots, t_{n-1})$;
- $content(t)$ is the set of elements that occur in t;
- $\varphi : \mathbb{N}^* \to \mathbb{N}$ is a learning function.

Identifiability in the Limit

Definition

Learning function φ:

1. identifies $S_i \in \mathcal{S}$ in the limit on t iff for co-finitely many m, $\varphi(t[m]) = i$;
2. identifies $S_i \in \mathcal{S}$ in the limit iff it identifies S_i in the limit on every t for S_i;
3. identifies $\mathcal{S}$ in the limit iff it identifies in the limit every $S_i \in \mathcal{S}$.

$\mathcal{S}$ is identifiable in the limit iff some learning function identifies $\mathcal{S}$ in the limit.

Example

Let $\mathcal{S}_1 = \{S_i \mid i \in \mathbb{N} - \{0\}\}$, where $S_n = \{1, \ldots, n\}$.

$\mathcal{S}_1$ is identifiable in the limit by the following function $\varphi : \mathbb{N}^* \to \mathbb{N}$:

$$\varphi(t[n]) = \max(content(t[n])).$$

Some Examples

Example

Let $\mathcal{S}_2 = \{S_i \mid i \in \mathbb{N}\}$, where $S_0 = \mathbb{N}$ and for $n \geq 1$, $S_n = \{1, \ldots, n\}$.

$\mathcal{S}_2$ is not identifiable in the limit.

Argument

To show that this is the case, let us assume that there is a function φ that identifies $\mathcal{S}_2$. We will construct a text, t on which φ fails:
t starts by enumerating $\mathbb{N}$ in order: $0, 1, 2, \ldots$
if at a number k learner φ decides it is S_0, t starts repeating k indefinitely.
This means t is a text for S_k.
As soon as φ decides it is S_k we continue with $k + 1, k + 2, \ldots$, so t will become a text for S_0, etc.
This shows that there is a text for a set from $\mathcal{S}_2$ on which φ fails.

Some Examples

Example

Let $\mathcal{S}_4 = \{S_n \mid S_n = \mathbb{N} - \{n\}, n \in \mathbb{N}\}$.

$\mathcal{S}_4$ is identifiable in the limit by the learning function $\varphi : \mathbb{N}^* \to \mathbb{N}$:

$$\varphi(t[n]) = \min(\mathbb{N} - content(t[n])).$$

Gold's Theorems

Theorem (1)

The class of all finite languages is identifiable.

Theorem (2)

The class containing all finite and at least one infinite language is not identifiable.

Cognitive Controversy

Gold's 2nd theorem

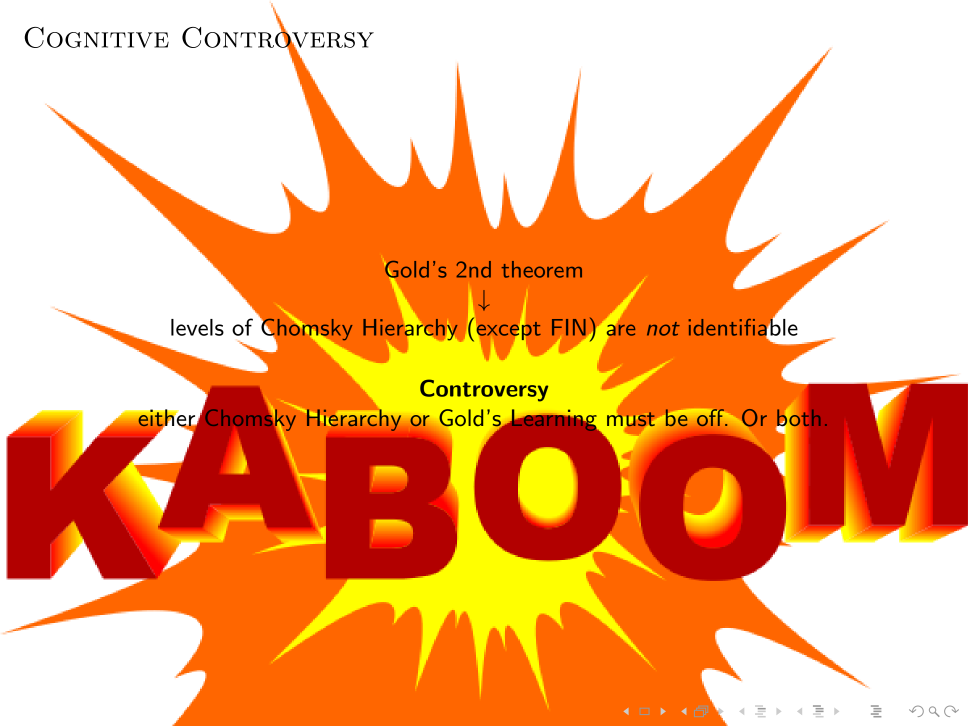
$\downarrow$

levels of Chomsky Hierarchy (except FIN) are *not* identifiable

Controversy

either Chomsky Hierarchy or Gold's Learning must be off. Or both.

Cognitive Controversy



Gold's 2nd theorem

$\downarrow$

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Controversy

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Some Restrictions on Learners

Learner φ is:

- **effective** if φ is a recursive function.
- **consistent** if, for each σ, $content(\sigma) \subseteq S_{\varphi(\sigma)}$.

  If φ is a consistent, recursive learner and φ identifies S, then S is recursive.
- **incremental** if, for all σ, τ, x, if $\varphi(\sigma) = \varphi(\tau)$, then $\varphi(\sigma^\wedge\langle x\rangle) = \varphi(\tau^\wedge\langle x\rangle)$.

  There is identifiable $\mathcal{S}$ that is not identifiable by an incremental learner.

  This can be overcome by fat text.
- **conservative** if, for all σ, x, $content(\sigma^\wedge\langle x\rangle) \subseteq S_{\varphi(\sigma)}$ implies $\varphi(\sigma^\wedge\langle x\rangle) = \varphi(\sigma)$.

  There is an effectively id. $\mathcal{S}$, but not by an effective conservative learner.

Limitting knowledge

True, there are good reasons for preferring the computable way of deriving knowledge. We know the results of computations and only think we know the results of trial and error procedures. There are many reasons for preferring knowing to thinking (as Popper, 1966, observed). But that does not change the fact that sometimes thinking may be more appropriate.

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In the next lecture
we will see how to model the dynamics of knowledge
via a combination of inductive inference and topology.

THANK YOU (and see you again tomorrow at 9am)