Multi-Agent Dialogue Games and Dialogue Sequents

PhDs in Logic 2016

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- Lorenzen-Dialogues as flexible reasoning procedure for **Intuitionistic** and **Modal Logic**
  - games and strategies

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  - two **parties**, more players
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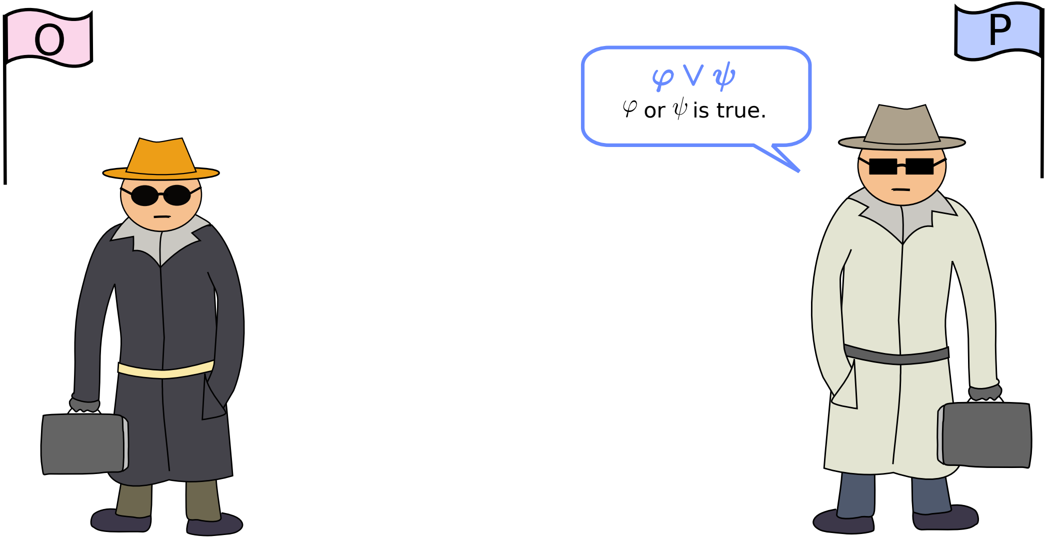
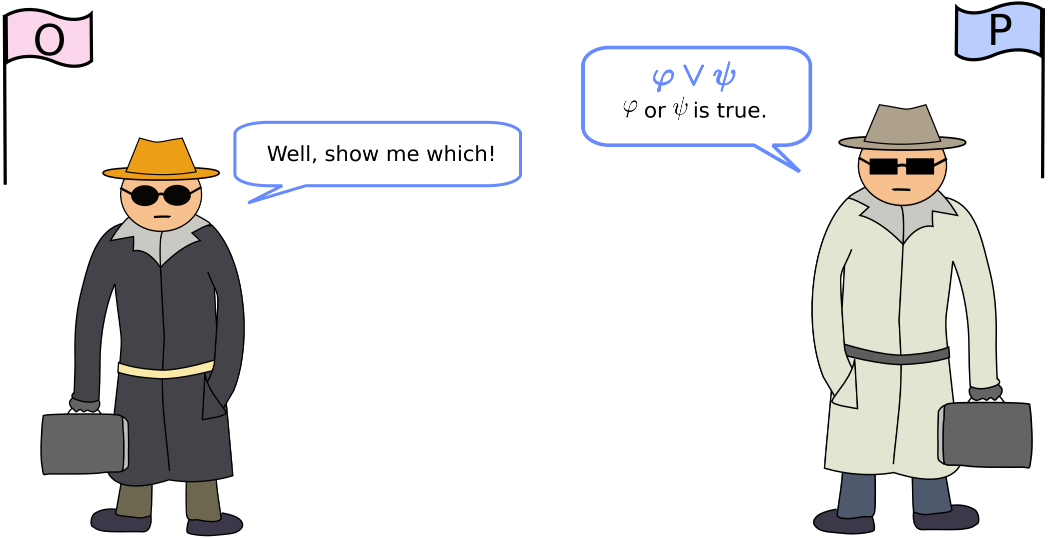
  - round-based scheduling
  - **normalization** simplifies proof-search
- Dialogue **Sequents**
  - clear rules, no ambiguity
  - easier to show soundness/completeness

O
P


O
P
$\varphi \vee \psi$
φ or ψ is true.



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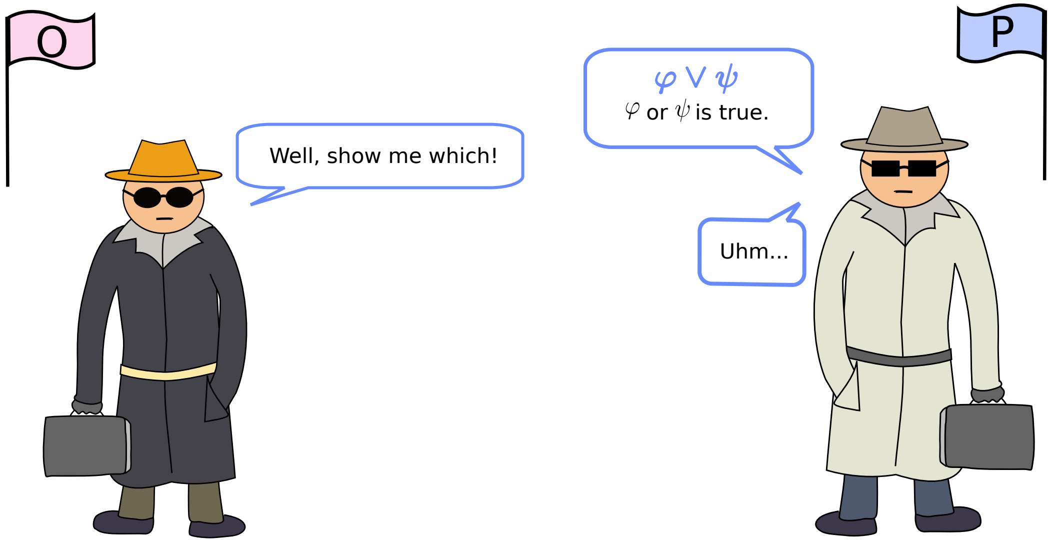
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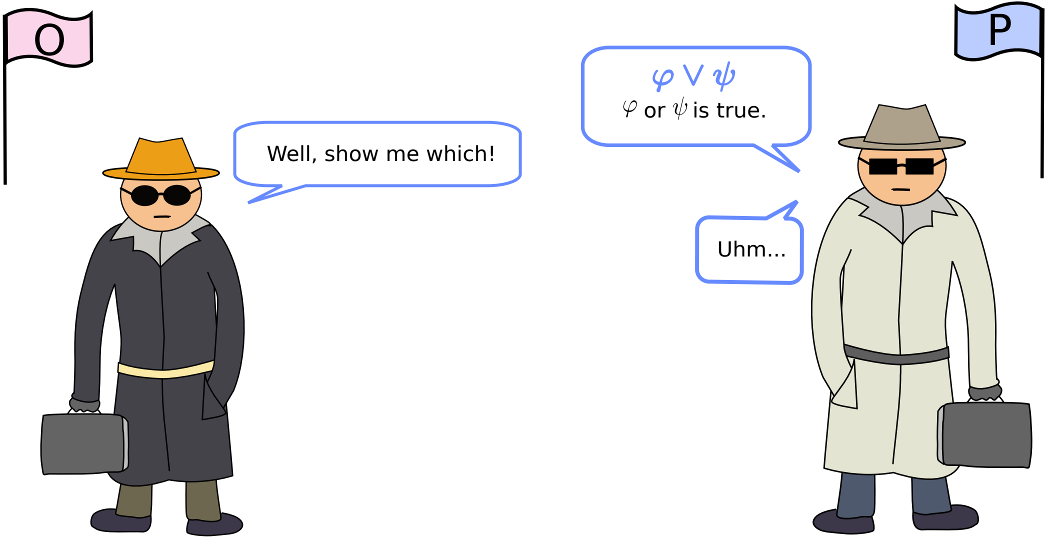
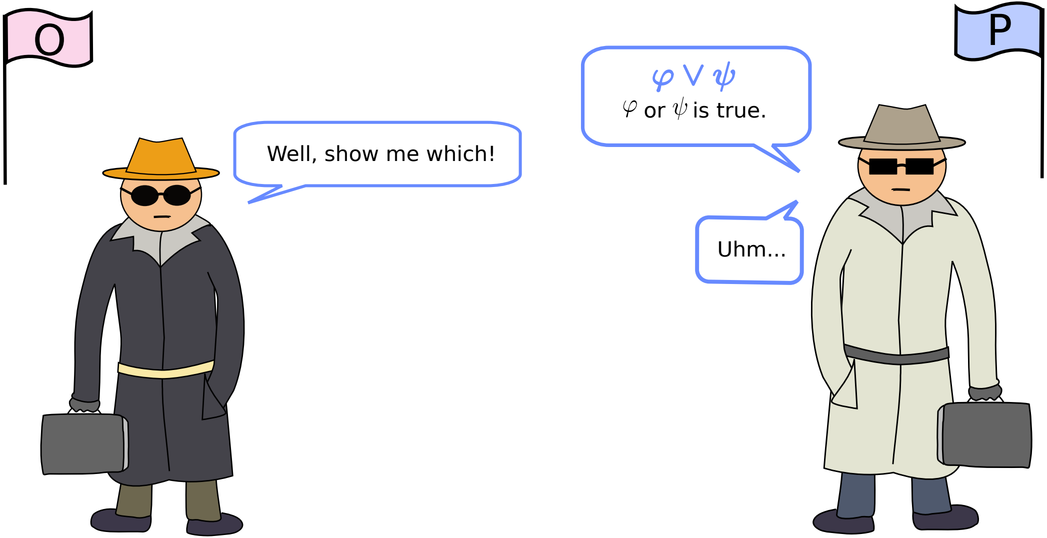
Well, show me which!
Uhm...



- P must decide whether to defend with φ or ψ.



- P must decide whether to defend with φ or ψ.
- In classical logic, he can defend more than once.

	O	P

	O		P	
1			**H**	$(A \wedge B) \supset (A \supset B)$

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4	B**?** (3)	$\lhd$		

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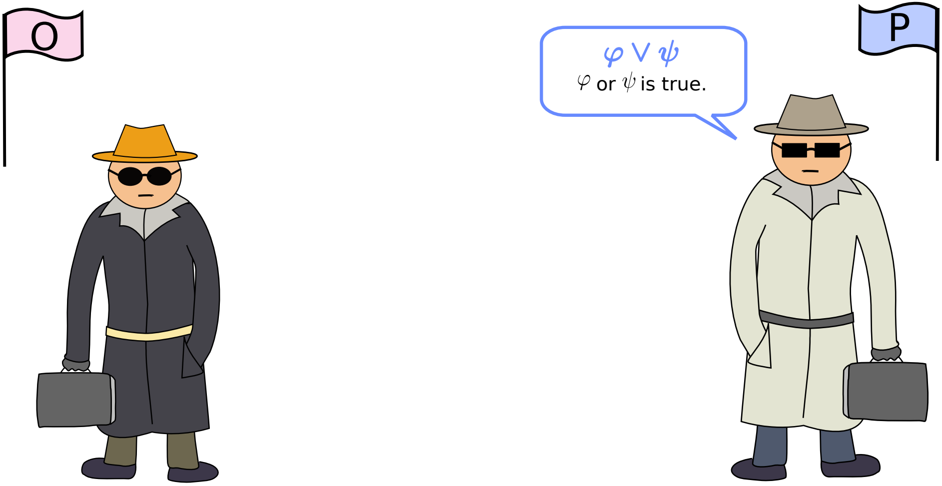
6	**!** (5)	B	**!** (4)	$\lhd$

- O may **attack atoms**, P may not.
- P may defend atom-attacks only if O stated atom herself (***ipse dixisti!***).
- In **intuitionistic dialogues**, P may only defend against the **last open attack**.


O
P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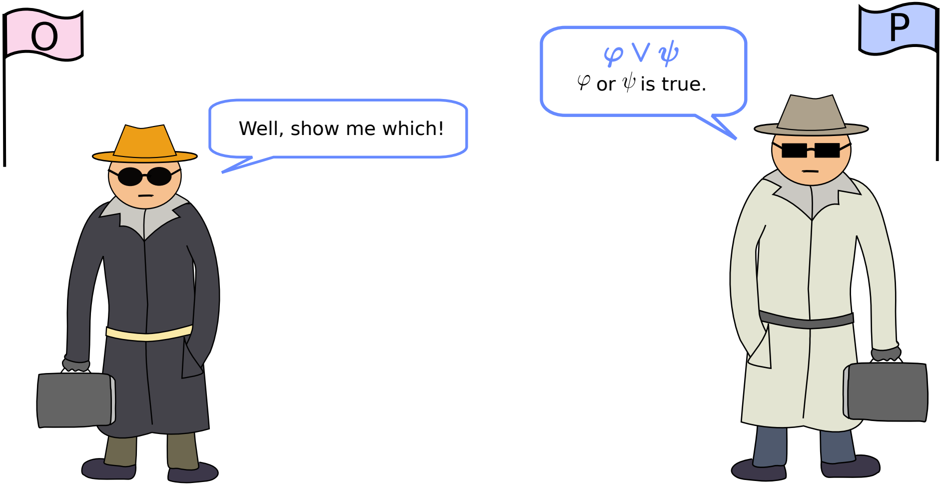
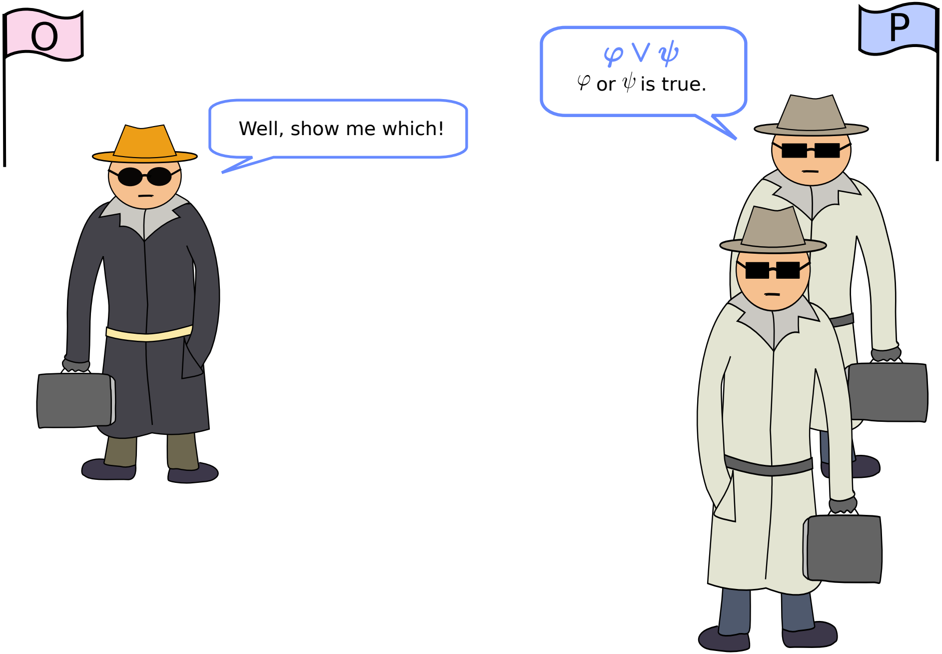

O
Well, show me which!
$\varphi \vee \psi$
φ or ψ is true.
P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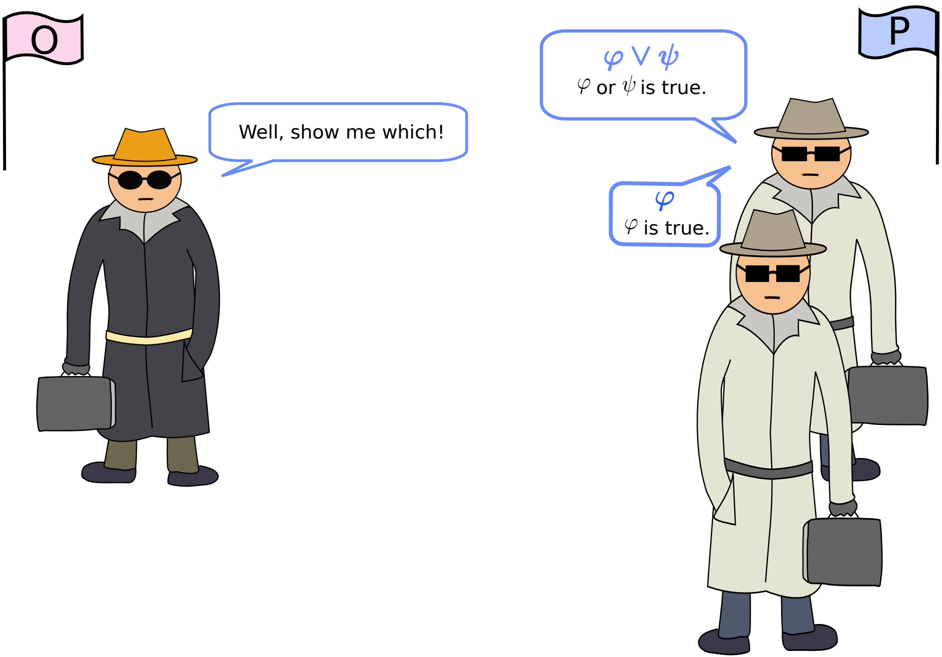
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φ or ψ is true.
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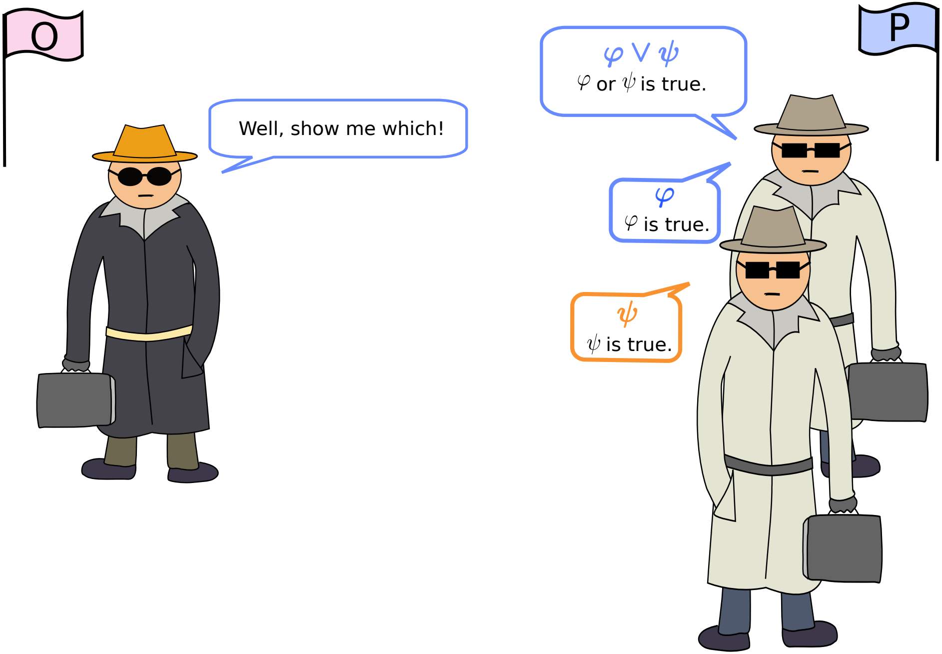
φ
φ is true.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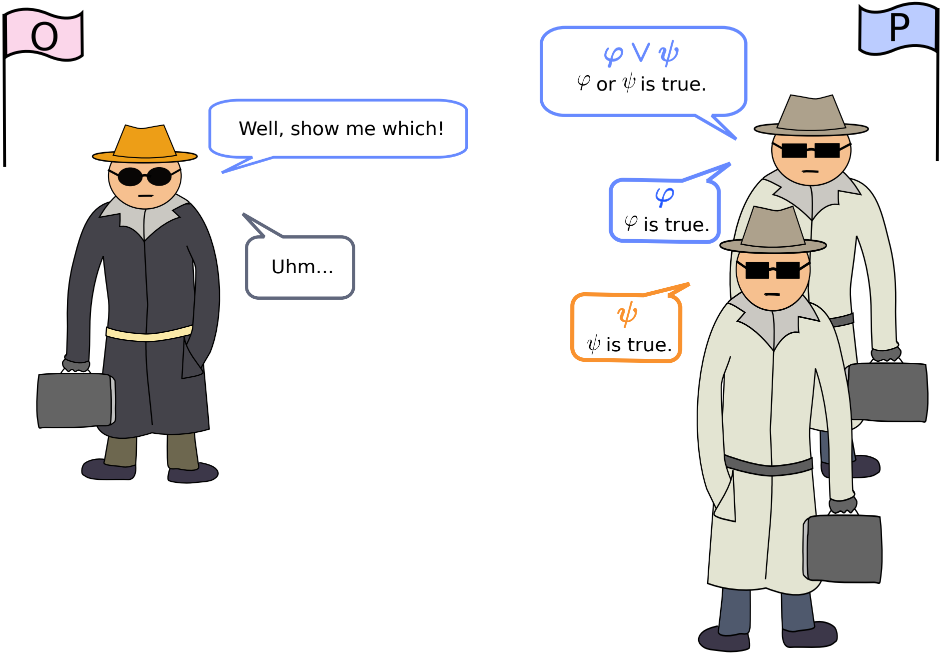
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φ
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Uhm...
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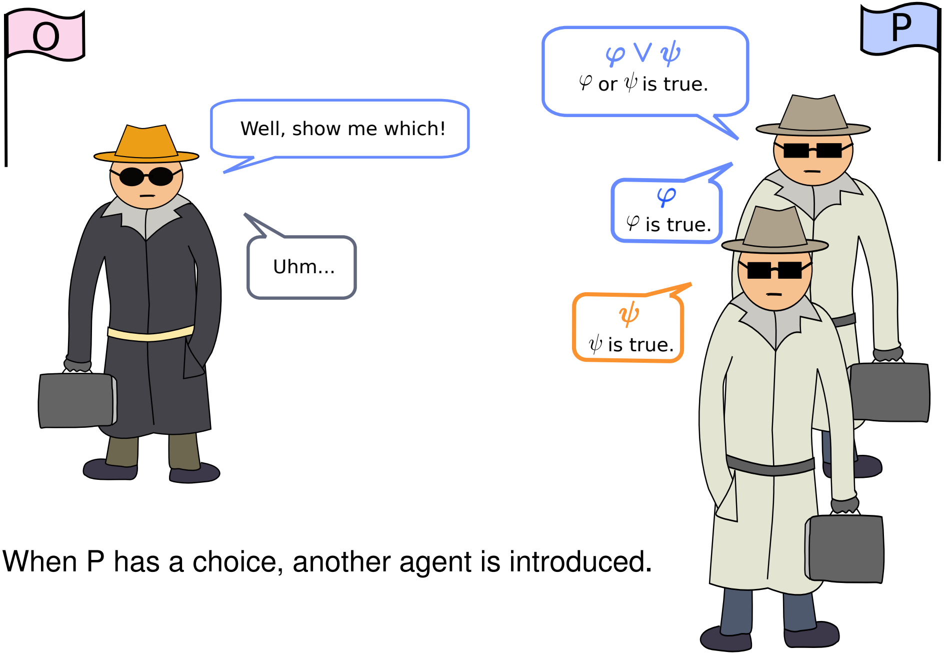
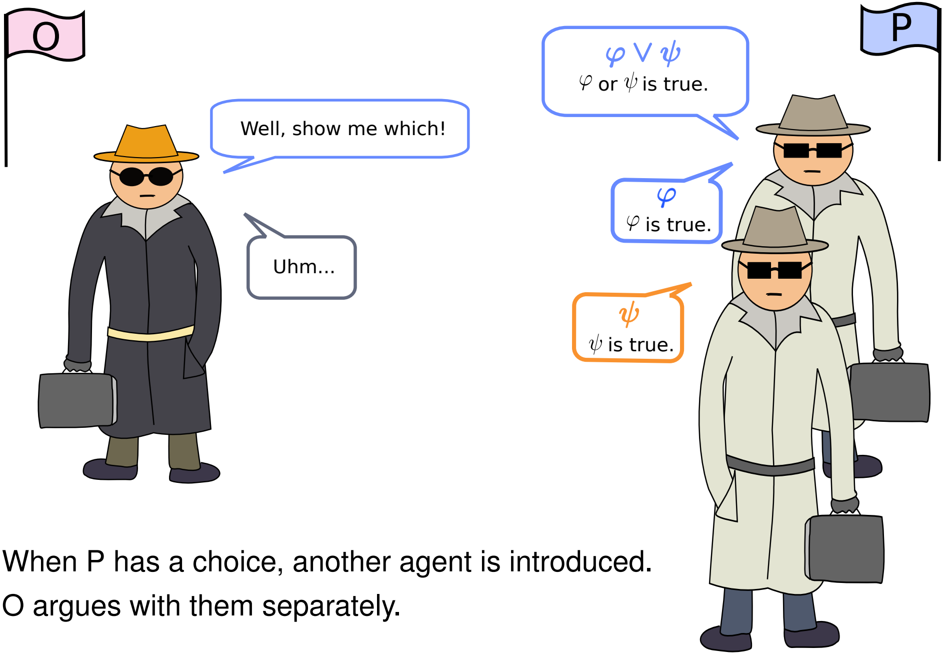


- When P has a choice, another agent is introduced.



- When P has a choice, another agent is introduced.
- O argues with them separately.

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	$!_{P1}$	A						

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  - Last moving party wins.

- Shared context and **intuitionistic** logic

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Distinguish between **critical** and **non-critical attacks**:

- critical: $\mathbf{?}\supset$ $\mathbf{?}\neg$
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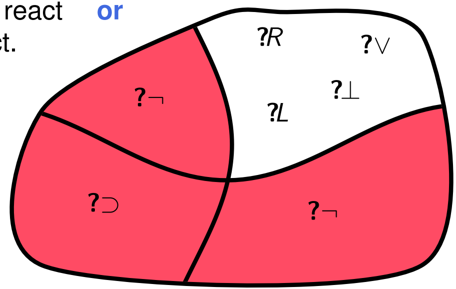

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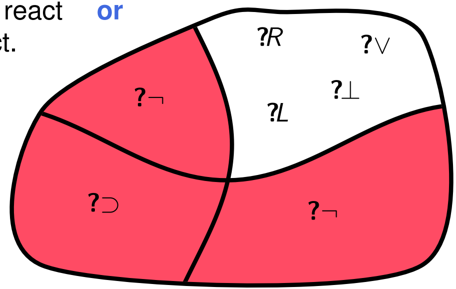
  - Critical defences may be delayed.



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  - $\widetilde{o_p} : \varphi$ assertion is **blocked** (optimization)

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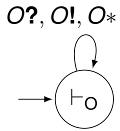

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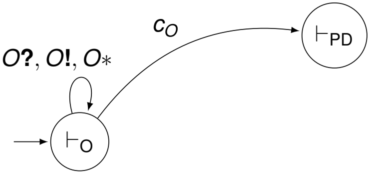

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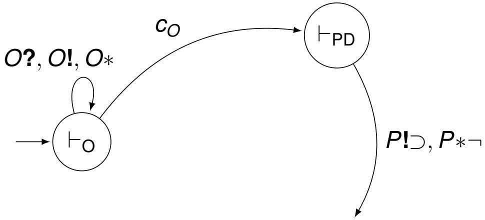

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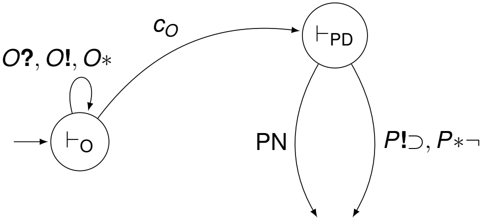

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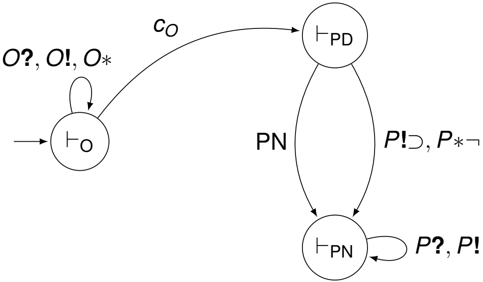
  P-agents performs their moves
  - attack O's assertions
  - defends **non-critically** against attacks



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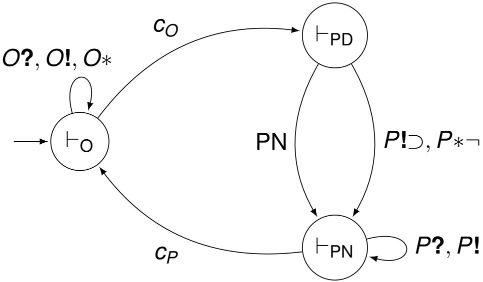

- Sequent Rules for O

$$\frac{\Gamma \vdash_O \Delta, \overline{p} : A \supset B}{\Gamma \vdash_O \Delta, p : A \supset B}\ O\mathbf{?}{\supset} \qquad \frac{\widetilde{o_p} : A \supset B, \Gamma \vdash_O \Delta, p : A \qquad o_p : B, \Gamma \vdash_O \Delta}{\overline{o_p} : A \supset B, \Gamma \vdash_O \Delta}\ O{*}{\supset}$$

$$\frac{\Gamma \vdash_O \Delta, \overline{p}^L : A \wedge B \qquad \Gamma \vdash_O \Delta, \overline{p}^R : A \wedge B}{\Gamma \vdash_O \Delta, p : A \wedge B}\ O\mathbf{?}{\wedge} \qquad \frac{\Gamma \vdash_O \Delta, \overline{p} : A \vee B}{\Gamma \vdash_O \Delta, p : A \vee B}\ O\mathbf{?}{\vee}$$

$$\frac{o_p : A, \Gamma \vdash_O \Delta}{\overline{o_p}^L : A \wedge B, \Gamma \vdash_O \Delta}\ O\mathbf{!}L \qquad \frac{o_p : A, \Gamma \vdash_O \Delta \qquad o_p : B, \Gamma \vdash_O \Delta}{\overline{o_p} : A \vee B, \Gamma \vdash_O \Delta}\ O\mathbf{!}{\vee}$$

$$\frac{o_p : B, \Gamma \vdash_O \Delta}{\overline{o_p}^R : A \wedge B, \Gamma \vdash_O \Delta}\ O\mathbf{!}R \qquad \frac{\Gamma \vdash_O \Delta, \overline{p} : \neg A}{\Gamma \vdash_O \Delta, p : \neg A}\ O\mathbf{?}{\neg} \qquad \frac{\widetilde{o_p} : \neg A, \Gamma \vdash_O \Delta, p : A}{\overline{o_p} : \neg A, \Gamma \vdash_O \Delta}\ O{*}{\neg}$$

$$\frac{\Gamma \vdash_O \Delta, \overline{p} : A}{\Gamma \vdash_O \Delta, p : A}\ O\mathbf{?}a \qquad \frac{\Gamma \vdash_O \Delta}{\Gamma \vdash_O \Delta, p : \bot}\ O\mathbf{?}{\bot} \qquad \frac{\Gamma \vdash_{PD} \Delta}{\Gamma \vdash_O \Delta}\ c_O$$

only applicable if no other rule application is possible

Sequent Rules for P-agents

p-rules – decide phase

$$\frac{o_q : A,\ \Gamma^{\delta} \vdash_{PN}\ p : B}{\Gamma \vdash_{PD}\ \Delta,\ \overline{p} : A \supset B}\ P!\supset \qquad \frac{o_p : A,\ \Gamma^{\delta} \vdash_{PN}\ \emptyset}{\Gamma \vdash_{PD}\ \Delta,\ \overline{p} : \neg A}\ P{*}\neg \qquad \frac{\Gamma \vdash_{PN}\ \Delta}{\Gamma \vdash_{PD}\ \Delta}\ \mathrm{PN}$$

$$\Gamma^{\delta} =_{df} (\Gamma \setminus \{\widetilde{o_p} : f \mid p \in Agents, f \in Form\}) \cup \{o_p : f \mid \widetilde{o_p} : f \in \Gamma, p \in Agents, f \in Form\}$$

q is a new P-agent.

p-rules – normal phase

$$\frac{\overline{o_p} : A \supset B,\ \Gamma\ \vdash_{PN}\ \Delta}{o_p : A \supset B,\ \Gamma\ \vdash_{PN}\ \Delta}\ P?\supset \qquad \frac{\overline{o_p} : \neg A,\ \Gamma\ \vdash_{PN}\ \Delta}{o_p : \neg A,\ \Gamma\ \vdash_{PN}\ \Delta}\ P?\neg \qquad \frac{\Gamma\ \vdash_{PN}\ \Delta,\ p : A}{\Gamma\ \vdash_{PN}\ \Delta,\ \overline{p}^{L} : A \wedge B}\ P!L$$

$$\frac{\overline{o_p}^{L} : A \wedge B,\ \overline{o_q}^{R} : A \wedge B,\ \Gamma\ \vdash_{PN}\ \Delta}{o_p : A \wedge B,\ \Gamma\ \vdash_{PN}\ \Delta}\ P?\wedge \qquad \frac{\Gamma\ \vdash_{PN}\ \Delta,\ p : B}{\Gamma\ \vdash_{PN}\ \Delta,\ \overline{p}^{R} : A \wedge B}\ P!R$$

$$\frac{\overline{o_p} : A \vee B,\ \Gamma\ \vdash_{PN}\ \Delta}{o_p : A \vee B,\ \Gamma\ \vdash_{PN}\ \Delta}\ P?\vee \qquad \frac{\Gamma\ \vdash_{PN}\ \Delta,\ p : A,\ q : B}{\Gamma\ \vdash_{PN}\ \Delta,\ \overline{p} : A \vee B}\ P!\vee$$

$$\frac{}{o_p : A,\ \Gamma\ \vdash_{PN}\ \Delta,\ \overline{s} : A}\ P!! \qquad \frac{}{o_p : \bot,\ \Gamma\ \vdash_{PN}\ \Delta,\ s : A}\ P?\bot \qquad \frac{\Gamma\ \vdash_{O}\ \Delta}{\Gamma\ \vdash_{PN}\ \Delta}\ c_P$$

only applicable if no other rule application is possible

q is a new P-agents in each case.

$$\emptyset \vdash_{\mathrm{O}} p0 : A \vee \neg A$$

$$\frac{\emptyset \vdash_{\mathsf{O}} \overline{p0} : A \vee \neg A}{\emptyset \vdash_{\mathsf{O}} p0 : A \vee \neg A}\ O\mathbf{?}\vee$$

$$
\dfrac{\dfrac{\dfrac{\emptyset \vdash_{\mathrm{PN}} \overline{p0} : A \vee \neg A}{\emptyset \vdash_{\mathrm{PD}} \overline{p0} : A \vee \neg A}\,\mathrm{PN}}{\emptyset \vdash_{\mathrm{O}} \overline{p0} : A \vee \neg A}\,c_O}{\emptyset \vdash_{\mathrm{O}} p0 : A \vee \neg A}\,O\mathbf{?}\vee
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\emptyset \vdash_{\mathsf{PN}} p0 : A,\; p1 : \neg A}{\emptyset \vdash_{\mathsf{PN}} \overline{p0} : A \vee \neg A}\ P\mathbf{!}\vee}{\emptyset \vdash_{\mathsf{PD}} \overline{p0} : A \vee \neg A}\ \mathsf{PN}}{\emptyset \vdash_{\mathsf{O}} \overline{p0} : A \vee \neg A}\ c_O}{\emptyset \vdash_{\mathsf{O}} p0 : A \vee \neg A}\ O\mathbf{?}\vee
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\emptyset \vdash_{\mathrm{O}} \overline{p0} : A,\ p1 : \neg A}{\emptyset \vdash_{\mathrm{O}} p0 : A,\ p1 : \neg A}\ O\mathbf{?}a}{\emptyset \vdash_{\mathrm{PN}} p0 : A,\ p1 : \neg A}\ c_P}{\emptyset \vdash_{\mathrm{PN}} \overline{p0} : A \vee \neg A}\ P\mathbf{!}\vee}{\emptyset \vdash_{\mathrm{PD}} \overline{p0} : A \vee \neg A}\ \mathrm{PN}}{\emptyset \vdash_{\mathrm{O}} \overline{p0} : A \vee \neg A}\ c_O}{\emptyset \vdash_{\mathrm{O}} p0 : A \vee \neg A}\ O\mathbf{?}\vee
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\emptyset \vdash_{\mathrm{O}} \overline{p0} : A,\ \overline{p1} : \neg A}{\emptyset \vdash_{\mathrm{O}} \overline{p0} : A,\ p1 : \neg A}\ O\mathbf{?}\neg}{\emptyset \vdash_{\mathrm{O}} p0 : A,\ p1 : \neg A}\ O\mathbf{?}a}{\emptyset \vdash_{\mathrm{PN}} p0 : A,\ p1 : \neg A}\ c_P}{\emptyset \vdash_{\mathrm{PN}} \overline{p0} : A \vee \neg A}\ P\mathbf{!}\vee}{\emptyset \vdash_{\mathrm{PD}} \overline{p0} : A \vee \neg A}\ \mathrm{PN}}{\emptyset \vdash_{\mathrm{O}} \overline{p0} : A \vee \neg A}\ c_O}{\emptyset \vdash_{\mathrm{O}} p0 : A \vee \neg A}\ O\mathbf{?}\vee
$$

$$
\begin{array}{cl}
\emptyset \vdash_{\mathrm{PD}} \overline{p0} : A,\ \overline{p1} : \neg A & \\ \hline
\emptyset \vdash_{\mathrm{O}} \overline{p0} : A,\ \overline{p1} : \neg A & c_O \\ \hline
\emptyset \vdash_{\mathrm{O}} \overline{p0} : A,\ p1 : \neg A & O\mathbf{?}\neg \\ \hline
\emptyset \vdash_{\mathrm{O}} p0 : A,\ p1 : \neg A & O\mathbf{?}a \\ \hline
\emptyset \vdash_{\mathrm{PN}} p0 : A,\ p1 : \neg A & c_P \\ \hline
\emptyset \vdash_{\mathrm{PN}} \overline{p0} : A \vee \neg A & P\mathbf{!}\vee \\ \hline
\emptyset \vdash_{\mathrm{PD}} \overline{p0} : A \vee \neg A & \mathrm{PN} \\ \hline
\emptyset \vdash_{\mathrm{O}} \overline{p0} : A \vee \neg A & c_O \\ \hline
\emptyset \vdash_{\mathrm{O}} p0 : A \vee \neg A & O\mathbf{?}\vee
\end{array}
$$

$$
\begin{array}{c}
o_{p1} : A \vdash_{\mathrm{PN}} \emptyset \\
\hline
\emptyset \vdash_{\mathrm{PD}} \overline{p0} : A,\ \overline{p1} : \neg A
\end{array}\ P{*}\neg
$$

$$
\begin{array}{c}
\emptyset \vdash_{\mathrm{PD}} \overline{p0} : A,\ \overline{p1} : \neg A \\
\hline
\emptyset \vdash_{\mathrm{O}} \overline{p0} : A,\ \overline{p1} : \neg A
\end{array}\ c_O
$$

$$
\begin{array}{c}
\emptyset \vdash_{\mathrm{O}} \overline{p0} : A,\ \overline{p1} : \neg A \\
\hline
\emptyset \vdash_{\mathrm{O}} \overline{p0} : A,\ p1 : \neg A
\end{array}\ O\mathbf{?}\neg
$$

$$
\begin{array}{c}
\emptyset \vdash_{\mathrm{O}} \overline{p0} : A,\ p1 : \neg A \\
\hline
\emptyset \vdash_{\mathrm{O}} p0 : A,\ p1 : \neg A
\end{array}\ O\mathbf{?}a
$$

$$
\begin{array}{c}
\emptyset \vdash_{\mathrm{O}} p0 : A,\ p1 : \neg A \\
\hline
\emptyset \vdash_{\mathrm{PN}} p0 : A,\ p1 : \neg A
\end{array}\ c_P
$$

$$
\begin{array}{c}
\emptyset \vdash_{\mathrm{PN}} p0 : A,\ p1 : \neg A \\
\hline
\emptyset \vdash_{\mathrm{PN}} \overline{p0} : A \vee \neg A
\end{array}\ P\mathbf{!}\vee
$$

$$
\begin{array}{c}
\emptyset \vdash_{\mathrm{PN}} \overline{p0} : A \vee \neg A \\
\hline
\emptyset \vdash_{\mathrm{PD}} \overline{p0} : A \vee \neg A
\end{array}\ \mathrm{PN}
$$

$$
\begin{array}{c}
\emptyset \vdash_{\mathrm{PD}} \overline{p0} : A \vee \neg A \\
\hline
\emptyset \vdash_{\mathrm{O}} \overline{p0} : A \vee \neg A
\end{array}\ c_O
$$

$$
\begin{array}{c}
\emptyset \vdash_{\mathrm{O}} \overline{p0} : A \vee \neg A \\
\hline
\emptyset \vdash_{\mathrm{O}} p0 : A \vee \neg A
\end{array}\ O\mathbf{?}\vee
$$

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  $\Rightarrow$ always same result wrt. P-winning-strategy

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$\Rightarrow$ proof normalization

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$\Rightarrow$ proof normalization

$\Rightarrow$ strategy analysis/optimization for Ps

Several agents on **O**-side?

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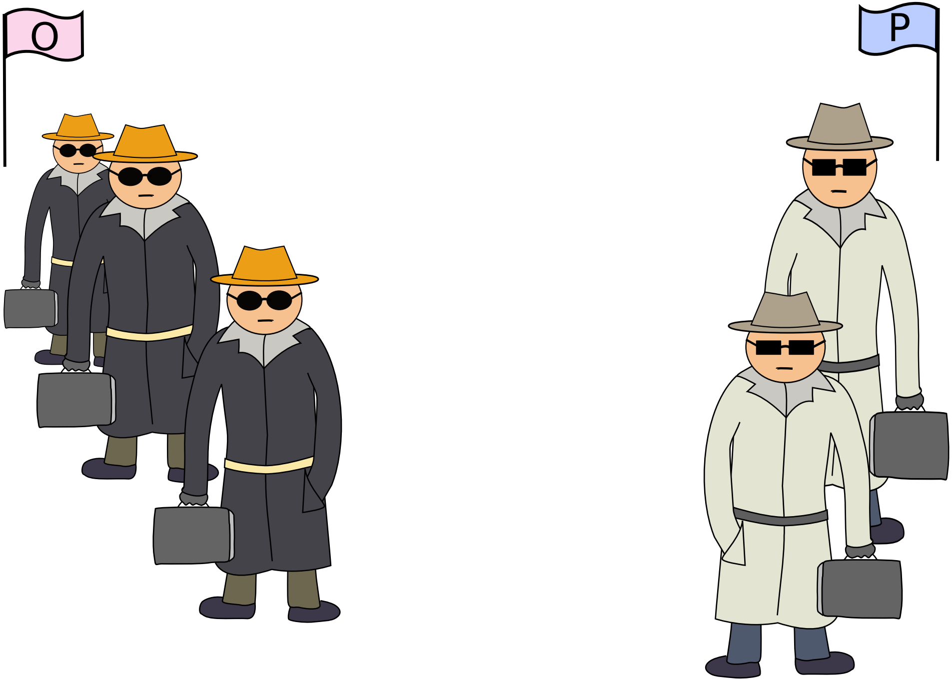
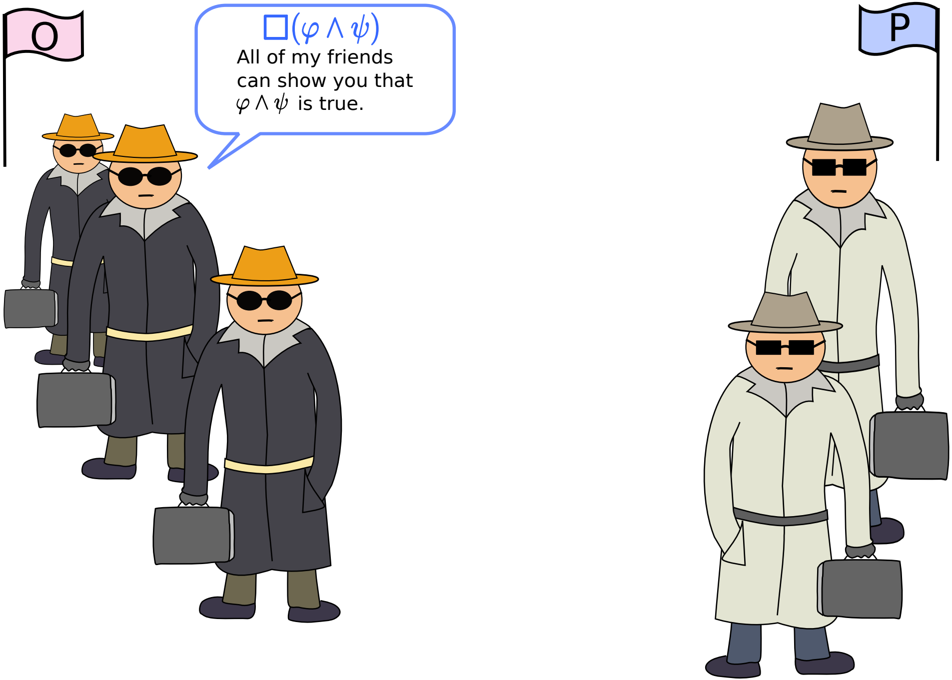
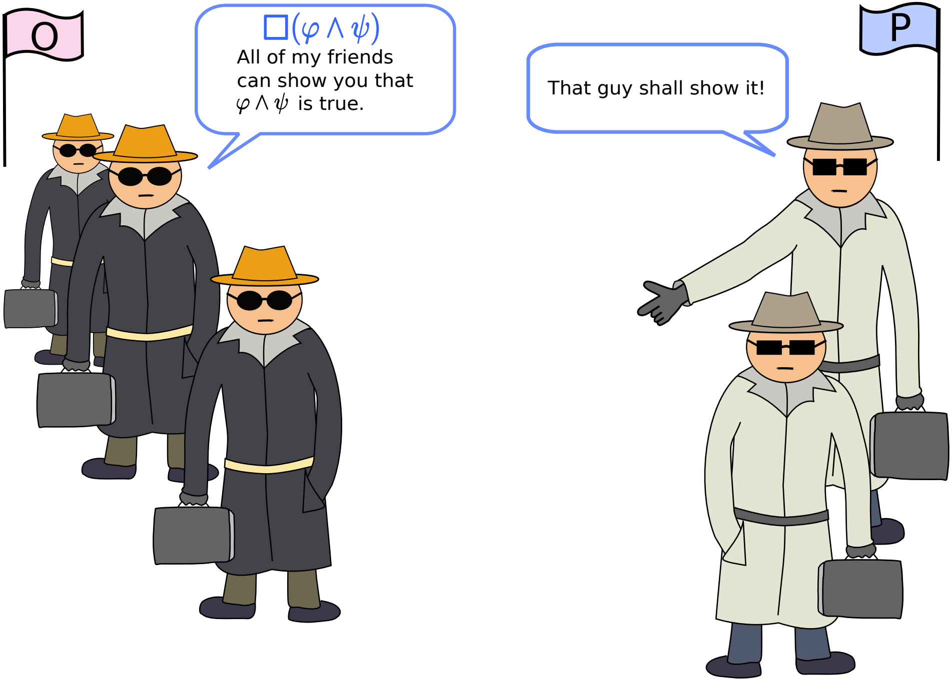
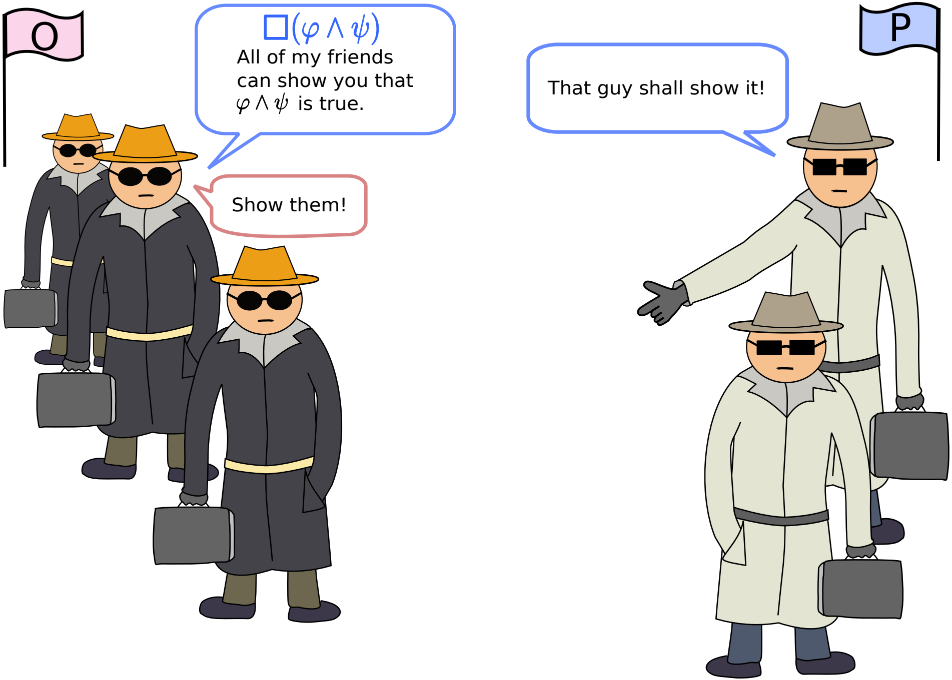
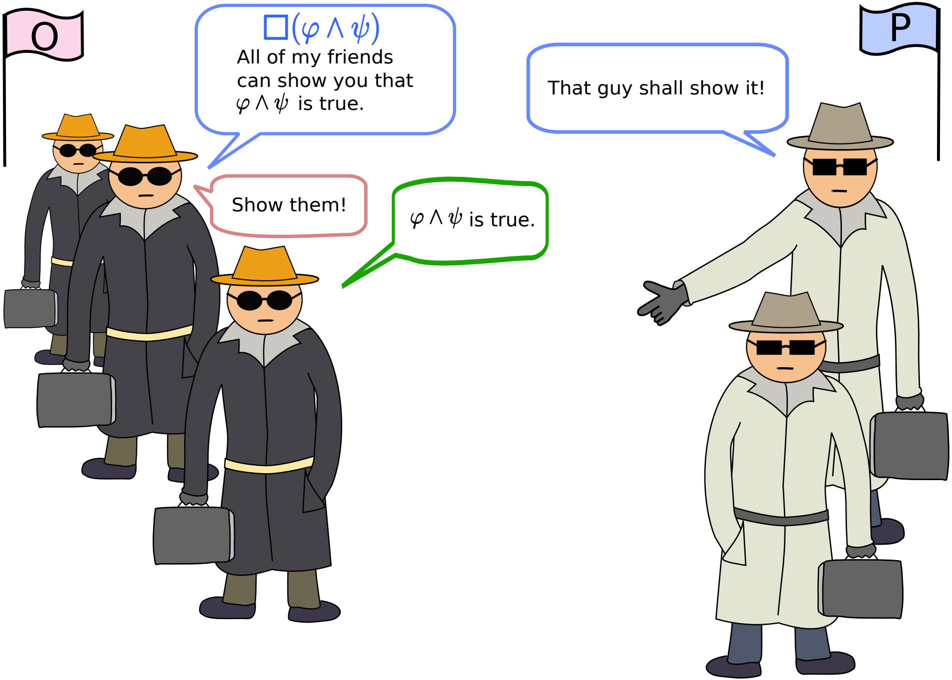
- Frank Van Dun (1972)
  $\Rightarrow$ **Modal Logic**
- O-agents correspond to Kripke worlds
  related due to **coalition relation** (usually reflexive)

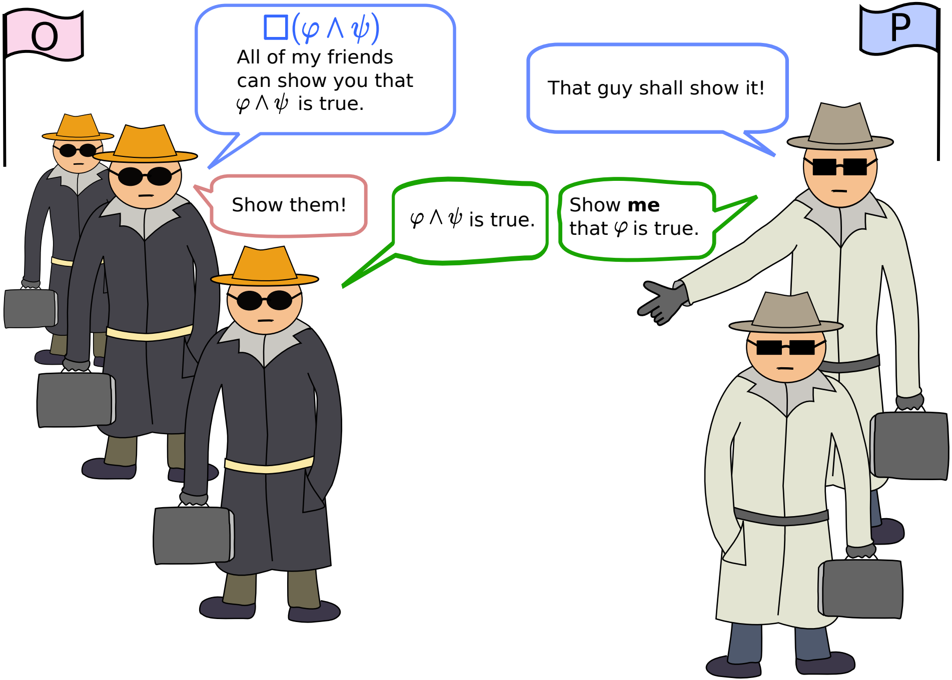

O
P

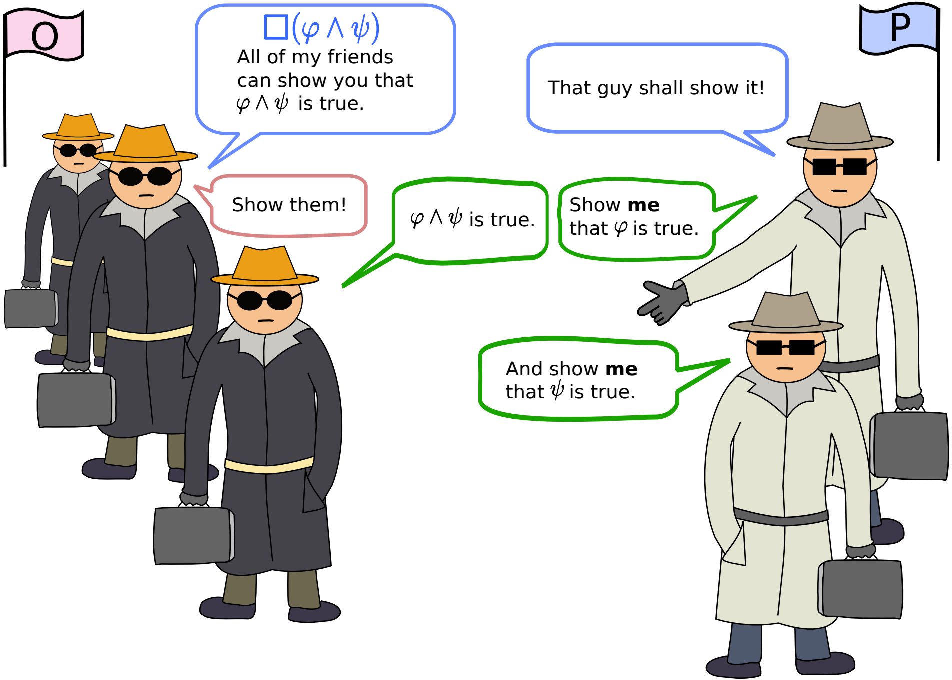

O
$\Box(\varphi \land \psi)$
All of my friends can show you that $\varphi \land \psi$ is true.
P

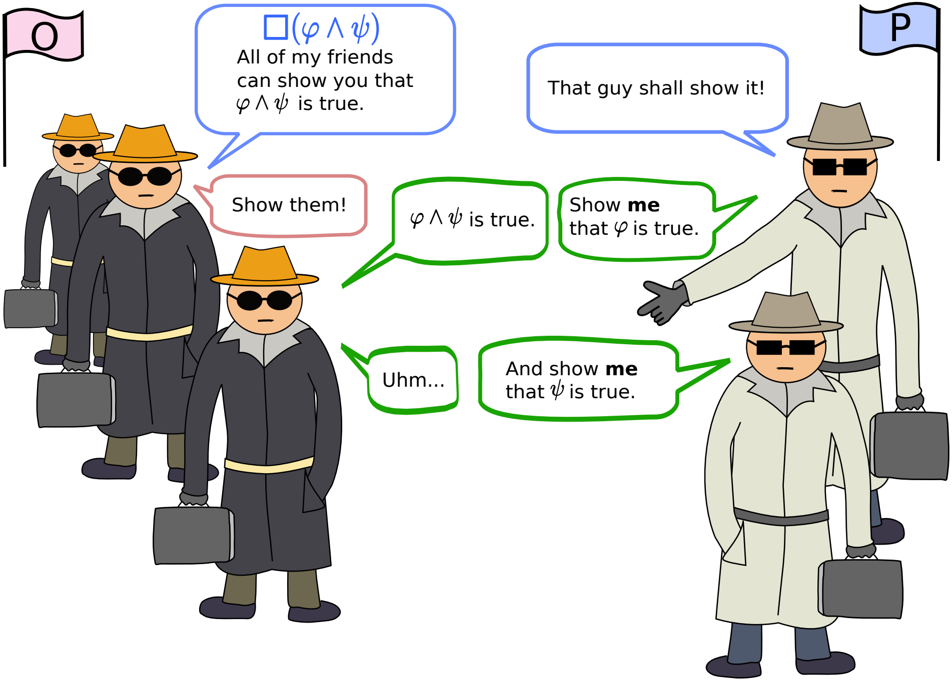

O
$\Box(\varphi \wedge \psi)$
All of my friends can show you that $\varphi \wedge \psi$ is true.
That guy shall show it!
P


O
$\Box(\varphi \wedge \psi)$
All of my friends can show you that $\varphi \wedge \psi$ is true.
Show them!
That guy shall show it!
P


O
$\square(\varphi \wedge \psi)$
All of my friends can show you that $\varphi \wedge \psi$ is true.
Show them!
$\varphi \wedge \psi$ is true.
P
That guy shall show it!


O
P
$\Box(\varphi \wedge \psi)$
All of my friends can show you that $\varphi \wedge \psi$ is true.
That guy shall show it!
Show them!
$\varphi \wedge \psi$ is true.
Show **me** that φ is true.


O
P
$\Box(\varphi \wedge \psi)$
All of my friends can show you that $\varphi \wedge \psi$ is true.
That guy shall show it!
Show them!
$\varphi \wedge \psi$ is true.
Show **me** that φ is true.
And show **me** that ψ is true.


O
P
$\Box(\varphi \wedge \psi)$
All of my friends can show you that $\varphi \wedge \psi$ is true.
That guy shall show it!
Show them!
$\varphi \wedge \psi$ is true.
Show **me** that φ is true.
Uhm...
And show **me** that ψ is true.

- Pairwise communication

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- O-agent states a **modal expression**

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    You choose!

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  - $\Diamond\varphi$ *At least* ***one*** *of my coalition partners can show you that* φ *is true.* ***I choose!***
- P-agent states a **modal expression**
  - $\Box\varphi$ *I can show that* φ *is true to* ***all*** *of your coalition partners.* ***You choose!***

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  - $\Box\varphi$ ***All*** *of my coalition partners can show you that* φ *is true.* ***You choose!***
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- Different coalition partners are bound to different **commitments**.

- Pairwise communication
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  - $\Box\varphi$ ***All*** *of my coalition partners can show you that φ is true.*
    You choose!
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- P-agent states a **modal expression**
  - $\Box\varphi$ *I can show that φ is true to* ***all*** *of your coalition partners.*
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    I choose!
- Different coalition partners are bound to different **commitments**.
- More under construction. . .
  S4, S5, IK . . .

Open questions / Future Work:

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- dialogical sequents for modal logic (sound/complete)

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- hybrid logic, public announcement

- Contact

martin.sticht@uni-bamberg.de