Abstracts for Talks at Domain VII

A. Simpson's Category of Topological Predomains
Ingo BATTENFELD (TU Darmstadt)

In recent work, Alex Simpson has discovered an interesting link between Classical Domain Theory and the realizability model over Scott's $\mathcal{P}\omega$ model for untyped λ-calculus.. In particular, Simpson uses the equivalence of $\mathbf{QCB}_0$, the category of quotients of countably-based T_0 spaces, and the category of extensional PERs over $\mathcal{P}\omega$, to define a category **TP** of *topological predomains*, which promises to satisfy certain requirements, such as supporting basic type constructors, recursive types, modelling polymorphism and free algebras for algebraic theories. Based on my Diplomthesis, I will show the above mentioned equivalence and gibe a definition of **TP**.

Synthetic Proof of the Kreisel-Lacombe-Shoenfiled Theorem
Andrej BAUER (Univ. Ljubljana)

The Graph semantics of λ-calculus, or,
The power set domain in its full lambda-glory
Chantale BERLINE (Univ. Paris VII, PPS)

A series of open questions on the semantics of (untyped) λ-calculus were proposed in [B], concerning the representability of extensions of λ-theories, the diverse semantics, including some old problems.

We will survey here some recent answers or progress brought by Salibra in a series of papers, some in collaboration with Bucciarelli or with myself, using graph models, and will also mention related problems which are still left open.

Graph models are the simplest models of (untyped) λ-calculus, and we advocated for long that they should be the first ones to consider for showing the consistency of extensions of λ-calculus (β-conversion), unless we have very good reasons not to do so. One can also use these models for proving some operational or syntactic properties of λ-calculus in a more economical and more fluid way than when using intersection type systems, and for getting information on the lattice of λ-theories.

Graph models can be viewed within Scott's continuous semantics, where they all have as underlying domain a full power set domain $\mathcal{P}(D)$, D infinite, (one could even restrict here to $\mathcal{P}(\mathbb{N})$). The class **GTh** of graph-theories (i.e. λ-theories of graph models), partially ordered by inclusion, is nevertheless very rich. To begin with we know since [Kerth 1994] that there is a continuum of pairwise (distinct and) incomparable, graph-theories.

The results presented here reveal part of the mystery about the structure of **GTh**, and of the class **SGTh** of all sensible graph models. Extending those results from **GTh** to **SGTh** ranges, when possible, from “straightforward” to “highly difficult” (case of the uncountability of **SGTh**).

There are two known ways of building graph models. By “canonical completion” of a “partial pair” which is essentially a recursive procedure, similar in essence to (non redundant) intersection type systems. The second one, called “forcing”, is highly non recursive. Let us define a “positive graph model” as a model which can be generated by a positive partial pair, in the sense of [B]. Positive graph models are sensible (the converse is false). In contrast models built by forcing are necessarily non sensible.

A key question of [B], which is still left open, concerns the existence of a (strong) approximation theorem allowing us to manage uniformly all the positive graph models. Thanks to Salibra [2004], who shows that **SGTh** admits BT as a top element, this would imply that BT is the unique possible λ-theory for a positive graph model. Partial positive results, which either cover distinct subclasses of models, or a signifiant subset of BT, and use different techniques, were obtained by Ronchi (private communication), Salibra, and Salibra-Berline (ongoing work). They justify retroactively why Kerth had to work with non positive pairs for building a continuum of graph theories, with, then, the difficult challenge to prove that enough such pairs, when well chosen, could however give distinct sensible theories. This was done in two successive steps by Kerth [1995] and David [1999].

[B] C. Berline *From computation to foundations: the λ-calculus and its webbed models*, TCS 249, p.81-161, 2000.

Sequentiality: A Survey

Pierre-Lois CURIEN (Univ. Paris VII, PPS)

Almost any domain is universal

Manfred DROSTE (TU Dresden)

Universal domains have been investigated by a number of authors since, as shown by D. Scott and G. Plotkin, they permit the definition of denotational semantics of programming languages. Scott [S] and Plotkin [P] gave explicit examples of a universal Scott-domain and a universal coherent domain, respectively. Gunter [G] described a universal bifinite domain, and Gunter and Jung [GJ] and Droste and Goebel [DG] gave, for different categories of domains, systematic methods of constructing universal objects which, by their additional property of saturation or homogeneity, even are unique up to isomorphism.

Here we wish to define in particular on the collection of ω-bifinite domains and of ω-Scott-domains, respectively, a (natural) structure of a probability space. Then we will show that in these spaces the collection of all universal domains has measure 1, thereby showing that 'almost any' bifinite (resp. Scott-) domain is universal. Our method also works for other classes of domains investigated in the literature.

In particular, we explicitly describe how to construct the universal homogeneous ω-bifinite domain and the universal homogeneous ω-Scott-domain probabilistically. Clearly, the structure of each domain is completely described by the structure of its subposet of compact elements. Since we always deal with ω-domains, we assume that the underlying set of compact elements is the set of natural numbers $\mathbb{N}$. Thus a domain is completely described by the order relation (of the domain) on the set $\mathbb{N}$. We provide a probabilistic construction of this partial order on the sets $1, ..., n$, successively for $n = 1, 2, ...$, thereby obtaining 'finite approximations' of the ultimate order on $\mathbb{N}$. When extending the order from $1, ..., n$ to $1, ..., n+1$, we have to decide probabilistically and fairly on the order relation of the 'new' element $n+1$ to the other elements. This is done subject only to the condition that certain already prescribed minimal upper bounds of subsets are preserved, which can be checked by a finite procedure. By this construction, the class of all bifinite domains with $\mathbb{N}$ as underlying set of compact elements becomes, in a natural way, a probability space.

Then we obtain:

Thm. 1. With probability 1, the described construction produces a universal homogeneous bifinite domain.

As a consequence we obtain:

Cor. 1. In the probability space of bifinite domains, the set of universal bifinite domains has measure 1, the set of domains which are embeddable (via an embedding-projection pair) into their continuous function space has measure 1, but the set of domains isomorphic to their function space has measure 0.

We also give such a probabilistic construction on $\mathbb{N}$ for the class of Scott-domains, and we obtain the analogous results for Scott-domains as **Thm. 1** and **Cor. 1**. Here, the order-theoretic considerations are a bit more complicated, but still manageable. As a consequence of our methods, we also obtain an explicit deterministic procedure for the construction of such universal domains (resp. their order of compact elements) on $\mathbb{N}$.

Joint work with Dietrich Kuske (Dresden).

Inverse and Implicit Function Theorems:
A Domain-theoretic Treatment

Abbas EDALAT (Imperial College, London)

We present a domain-theoretic version of the inverse and implicit function theorems, using the recently constructed domain for multi-variable calculus. We obtain a refinement of the inverse function and the implicit function theorems for Lipschitz functions in classical analysis and show that these functions can be obtained as the sup of an increasing sequence of step functions if the original function is given as the sup of an increasing sequence of step functions. For differentiable functions, we show that the inverse and implicit functions can also be obtained as the sup of an increasing sequence of step functions if the derivative of the original function is given as the sup of an increasing sequence of step functions.

This is joint work with Andre Lieutier and Dirk Pattinson

Two right topologies for spectral theory of semitopological semilattices

Yu. L. ERSHOV (Sib. Branch of the Russian Academy of Science, Novosibirsk)

Finitary approximations of Markov processes

M. ESCARDÓ (Univ. Birmingham) (joint work with V. Danos (Paris VII))

Computational issues regarding continuous-state (labelled or not) Markov processes lead one to consider "discretizations", or finitary approximations, of them. One approach, developed by Danos, Desharnais, and Panangaden (2003), considers certain approximants that are not themselves Markov processes, but instead certain "pre-Markov processes", where probabilities are replaced by "uncertain probabilities".

Their formalization of these ideas takes place in the category of measurable maps of measure spaces. Given a measure space X, one first forms a measure space $\mathcal{M}\,X$ of (sub)probability measures on X, and a Markov process with state space X is defined to be a measurable map $X \to \mathcal{M}\,X$. A pre-Markov process is a quotient of a Markov process, obtained by considering a finitely generated sub-σ-algebra, called a pixelization of X. Two points of X are defined to be equivalent if they live in the same events of the pixelization. The restriction of a probability measure to the quotient is no longer a probability measure, but it can be regarded as an uncertain probability measure. By taking finer and finer finitary quotients, Markov processes and probability measures arise as limits of their degraded versions. A major complication of this approach is that maps involving the quotient construction are hardly ever measurable, or at least difficult check to be measurable. The above authors side-step this obstacle to some extent by considering quotients induced by sets of formulas of a Milner-Henessy type logic, where the complication doesn't arise, and this is good enough for many purposes. One could further criticize this approximation theory for not giving an accompanying notion of extended integration, and therefore not making clear how one calculates average behaviours of an approximant.

It is clear that the above ideas have a domain-theoretic flavour. However, domain theory cannot be applied directly to their formalization via measure spaces, mainly due to the generality of the framework. In order to be able to take advantage of the domain-theoretic machinery, we restrict from measure spaces to (certain) topological spaces, which will accomodate both state spaces (typically Hausdorff) and spaces of approximate probability measures (always non-Hausdorff), and from measurable maps to continuous maps. Of course, from a computational perspective, this is not a genuine restriction, as domains of computation come naturally equipped with topologies induced from notions of observable property, and computable maps are automatically continuous with respect to these topologies. The spaces we consider include compact Hausdorff ones and a variety of continuous domains under the Scott topology (and they are precisely the so-called stably compact spaces).

In order to develop integration within domain theory for a compact Hausdorff space X, Edalat (1995) considered a domain environment $\tilde{X}$ for X, and then showed that $\mathcal{V}\tilde{X}$ serves as a domain environment for the space of Borel regular measures with the weak topology. Here $\mathcal{V}$ is the powerdomain construction introduced by Jones (1990) in order to study probabilistic computation. Thus, an uncertain probability measure on X could be articulated as a member of $\mathcal{V}\tilde{X}$, and a pre-Markov process with state space X could be articulated as a continuous map $X \to \mathcal{V}\tilde{X}$. From our perspective, there are two fundamental problems with this: (1) $\mathcal{V}\tilde{X}$ doesn't contain enough uncertain elements — we would like to be able to take arbitrary non-empty meets of precise measures, for the purpose of constructing quotients, and (2) the function spaces $(X \to \mathcal{V}\tilde{X})$ and $(\tilde{X} \to \mathcal{V}\tilde{X})$ both fail to form continuous domains and hence to support the idea of finitary approximation.

Interestingly, a solution to problem (1) automatically provides a solution to (2), because the continuous domains with non-empty meets (equivalently, bounded joins) form an exponential ideal in the category of topological spaces. That is, if X is exponentiable (for example, locally compact) and D is such a domain (with the Scott topology), then the exponential $(X \to D)$ is again such a domain. In particular, such domains, being themselves locally compact, form a cartesian closed category of spaces. On the other hand, no cartesian closed category of continuous domains closed under the probabilistic powerdomain is known to exist (Jung and Tix (1998)).

Recently, Alvarez-Manilla, Jung and Keimel (2004) showed that stably compact spaces are closed under the probabilistic powerdomain construction. In particular, for a compact Hausdorff space X, the space $\mathcal{V}X$ has a domain environment $\widetilde{\mathcal{V}X}$, for example its Smyth powerdomain, which is bounded complete and hence solves problems (1) and (2) above. (Cf. also recent work by Tix, Keimel and Plotkin (2004)). Thus, an uncertain measure can be articulated by a compact set of valuations and the resulting pre-Markov processes form a bounded complete continuous domain environment $(X \to \widetilde{\mathcal{V}X})$ for the function space $(X \to \mathcal{V}X)$ of Markov processes. (Here we use the fact that the embedding $\mathcal{V}X \to \widetilde{\mathcal{V}X}$ is proper and dense, that bounded complete continuous domain are densely injective spaces, and apply a theorem on densely injective spaces and function spaces by Escardó 1998). However, from the technical point of view, compact sets of valuations are of course not always easy to work with. Hence we propose another solution, which consists of constructing a bounded complete continuous domain $\tilde{\mathcal{V}}X$ of "pre-valuations".

We first construct the function space $\mathcal{T}X = ((X \to \mathbb{S}) \to \mathbb{I})$, where $\mathbb{S}$ is the Sierpinski space and $\mathbb{I}$ is the unit interval with the Scott topology of the natural order. This is a bounded complete continuous domain (in fact, continuous lattice), containing the continuous valuations as a subspace. It follows from domain theory that the topological closure of this subspace is again a bounded complete continuous domain, and we let this be the domain $\tilde{\mathcal{V}}X$ of pre-valuations. In this example, the topological closure turns out to be just the order-theoretic lower closure, and thus a pre-valuation is simply a continuous map below a valuation. This domain has a basis consisting of finite joins of "fat dirac distributions", i.e. pre-valuations of the form $t\delta_K$ with $t \in \mathbb{I}$ and $K \subseteq X$ compact non-empty, defined by $t\delta_K(U) = t$ if $K \subseteq U$ and 0 otherwise. The idea is that we have the partial information that an amount t of mass is distributed over K, but we don't know how it is distributed. We thus have a more concrete, alternative bounded complete continuous domain environment $(X \to \widetilde{\mathcal{V}}X)$ of pre-Markov processes for the space $(X \to \mathcal{V}X)$ of Markov processes. In particular, finitary approximants are easily described and constructed. This is perhaps the primary practical improvement of the approach that we propose.

Another improvement is that our approach automatically generalizes from discrete to continuous spaces of labels. For any locally compact space L, we can consider labelled pre-Markov processes $X \to (L \to \tilde{\mathcal{V}}X)$, which again form a bounded complete continuous domain. A typical situation arises when L is the real line, which in practice is discretized before modelling systems of interest, as the current theory based on measure spaces cannot cope with continuous label spaces. In our approach, there is no need to perform the discretization of the label space at the modelling stage.

¿From a theoretical point of view, it is now easy to introduce effectivity notions, and hence it is possible to study computability questions. Moreover, our notion of pre-valuation can perhaps be taken as a base for the design and semantics of a programming language for Markov processes in the style of Real PCF (Escardó 1996), which has the property that is not necessary to pre-discretize continuous systems before running them, provided they are expressible in the language.

Finally, we have developed integration with respect to pre-valuations, concretely using a Choquet-type integral, and abstractly using injectivity of the space $\mathbb{I}$, and shown the two approaches coincide. We have also looked at bisimulation and its logical characterization via a Milner-Henessy-type logic.

(We observe that many intermediate lemmas are obtained "for free" by an application of the synthetic topological methods of Escardó (2004).)

Algebraically labeled transition systems

Hans-Peter GUMM (Univ. Marburg)

Starting with a complete semilattice L or with a monoid M, we consider the functors $L^{(-)}$ (resp $M_\omega^{(-)}$) as type functors for coalgebras. These can be interpreted as transitions systems with labels from L, resp. M.

We describe the interplay between the algebraic structure of L, resp. M, with preservation properties of the functor, and with the structure theoretic properties of the associated coalgebras.

For instance, the monoid M is refinable iff $M\omega^{(-)}$ weakly preserves kernel pairs iff every coalgebra congruence is a bisimulation.

We finally generalize the setup by constructing coalgebraic type functors from more general structures.

A Cartesian Closed Category Containing the Category of Locales

R. HECKMANN (AbsInt Angewandte Informatik GmbH)

Summary: We present a Cartesian closed category ELoc of *equilocales*, which

contains the category Loc of locales as a reflective full subcategory. The embedding of Loc into ELoc preserves products and all exponentials of exponentiable locales.

More details: So far, no Cartesian-closed extension of the category Loc of locales was known. Here we present one such extension, called the category ELoc of *equilocales.* The new category has some similarity with the category of *equilogical spaces*, which is one of the Cartesian closed extensions of $\mathcal{T}_0$-Top. Recall that there are several equivalent categories of equilogical spaces of different kinds, for instance $\mathcal{T}_0$-topological spaces carrying an equivalence relation, or continuous lattices (= injective spaces) carrying a *partial* equivalence relation (PER). In a similar way, we present two different but equivalent categories of equilocales: the objects of IELoc involve an injective locale and a family of PERs, while the objects of SELoc involve an arbitrary locale and a family of PERs satisfying a joint fullness condition. For matters of economy, we first introduce a larger category ELoc^* whose objects involve an arbitrary locale and a family of PERs.

Note that a PER on a space X in Top, i.e., on the set of points of X, corresponds to a PER on the set $\mathsf{Top}\,(\mathbf{1}, X)$ of continuous functions from the terminal space (one-point space) $\mathbf{1}$ to X. Here, we replace the topological space X by a locale X, but we also need to get away from considering $\mathbf{1}$ since there are non-trivial locales X with no points ($\mathsf{Loc}\,(\mathbf{1}, X) = \emptyset$). The solution is to consider not only a PER on the single set $\mathsf{Loc}\,(\mathbf{1}, X)$, but a family of PERs consisting of one PER on each set $\mathsf{Loc}\,(S, X)$, for any locale S.

DEFINITION: A generalized equilocale (object of ELoc^*) $\mathcal{X}$ is a pair $(X, \sim_\mathcal{X})$ consisting of a locale $X = |\mathcal{X}|$ (the *target locale* of $\mathcal{X}$) and a family $\sim_\mathcal{X} = (\sim^S_\mathcal{X})_{S\in\mathsf{Loc}}$ where $\sim^S_\mathcal{X}$ is a PER on the set $\mathsf{Loc}\,(S, X)$ of locale maps from S to X, subject to the following compatibility condition: $\forall\, s : R \to S : \quad x \sim^S_\mathcal{X} x' \;\Rightarrow\; x\,s \sim^R_\mathcal{X} x's\,.$

DEFINITION: Given two generalized equilocales $\mathcal{X} = (X, \sim_\mathcal{X})$ and $\mathcal{Y} = (Y, \sim_\mathcal{Y})$, we define a PER '$\approx$' on the set $\mathsf{Loc}\,(X, Y)$ of locale maps from X to Y by $f \approx f'$ iff for all locales S, $x \sim^S_\mathcal{X} x'$ implies $fx \sim^S_\mathcal{Y} f'x'$. The set $\mathsf{ELoc}^*\,(\mathcal{X}, \mathcal{Y})$ of ELoc^* maps from $\mathcal{X}$ to $\mathcal{Y}$ is then defined as the set of partial equivalence classes $\mathsf{Loc}\,(X, Y)/{\approx}$.

An *in-equilocale* is a generalized equilocale $(A, \sim_\mathcal{A})$ whose target locale A is injective. The full subcategory IELoc of ELoc^* whose objects are in-equilocales is *Cartesian closed.*

A *sur-equilocale* is a generalized equilocale $\mathcal{X} = (X, \sim_\mathcal{X})$ such that the class of self-related $x : S \to X$ is jointly epi, i.e., $f\,x = f'\,x$ for all self-related x implies $f = f'$. The full subcategory SELoc of ELoc^* whose objects are sur-equilocales is *equivalent to* IELoc, hence Cartesian closed, too.

The category Loc of locales embeds into SELoc by mapping X to $\widehat{X} = (X, \sim_{\widehat{X}})$ with $x \sim^S_{\widehat{X}} x'$ iff $x = x'$. This embedding preserves products and exponentials Z^Y of exponentiable locales Y. (A locale Y is *exponentiable* if exponentials Z^Y exist for all locales Z.) Finally, we establish a reflection from SELoc to its subcategory Loc.

In showing these results, we never need to delve into the details of the internal structure of locales. We only need some general properties of these objects: products, equalizers, and coequalizers exist, every locale is a sublocale (regular subobject) of an injective locale, and the category of injective locales is Cartesian closed. Thus, our results hold in fact for categories different from Loc if only the required general properties are guaranteed.

A domain for refinement of modal transition systems

Michael HUTH (Imperial College, London)

We construct a countably based SFP domain D and prove it to be a fully abstract and universal model for Larsen & Thomsen's modal transition systems (joint work with Radha Jagadeesan and David Schmidt). We use a game semantics for refinement and the universality and full abstraction of D to prove that refinement between modal transition systems is nothing but reverse containment of implementations, where the latter are refining labeled transition systems and modeled as maximal elements in D. A finer analysis of the set $\max(D)$ of maximal elements of D reveals that $\max(D)$ is an accurate model of the class of labelled transition systems modulo bisimulation. In particular, this quotient is a Stone space in the topology induced by the Scott-topology of D. This leads to applications in measuring the degree of consistency of several modal transition systems that describe the same underlying or putative artifact.

Domain environments for real numbers

Achim JUNG (Univ. Birmingham)

A domain environment for a topological space X is a (usually continuous) dcpo D together with a homeomorphism from X to the space of maximal elements of D endowed with the relative Scott-topology. Through the work of Lawson, Edalat, Martin and others, quite a bit is now known about the kind of spaces which can be presented in this way. Another motivation for studying domain environments, however, is to provide a setting in which the elements of the space in question can be used in actual computation. The prime examples are Edalat's approach to computable measure theory and exact real number computation.

In this talk I will concentrate on the latter example and discuss the various domain environments that have been suggested for the real line (or a compact interval). Theorems by di Gianantonio, on the one hand, and Escardó, Hofmann, and Streicher, on the other hand, suggest that the choice of domain environment may have an impact on the set of functions that can be computed efficiently. I will argue that these limitations to efficiency are indeed not inherent in the real numbers but arise from the way domain environments interact with the programming language.

This is joint work with Amin Farjudian.

Constructing semantic domains by fixed ponits of self-maps

Hui KOU (Sichuan University)

Sober spaces: some old and some new
Jimmie LAWSON (Louisiana State Univ.)

Convex Powerdomains and Vietoris Spaces
Jihua LIANG (Sichuan Univ.)

A universal model for an infinitary CPS target language
Tobias LÖW (TU Darmstadt)

Domain theoretical models of linear polymorphism
Paola MANEGGIA (Darmstadt)

We give a construction to get a model of linear logic with second order universal quantification starting from a suitable linear category of domains (such as the category of prime algebraic lattices and functions preserving arbitrary suprema). Key ingredients are lax transformations, lax limits and bifiniteness. This construction also accounts for the Coquand-Gunter-Winskel model of System F based on Scott-domains.

Semantics of a sequential language
for exact real number computation
Jose Raymundo MARCIAL Romero (Univ. Birmingham)[1mm] We study a programming language with a built-in ground type for real numbers. In order for the language to be sufficiently expressive but still sequential, we consider a construction proposed by Boehm and Cartwright. The non-deterministic nature of the construction suggests the use of powerdomains in order to obtain a denotational semantics for the language. We show that the construction cannot be modelled by the Plotkin or Smyth powerdomains, but that the Hoare powerdomain gives a computationally adequate semantics. As is well known, Hoare semantics can be used in order to establish *partial* correctness only. Since computations on the reals are infinite, one cannot decompose total correctness into the conjunction of partial correctness and termination as it is traditionally done. We instead introduce a suitable operational notion of strong convergence and show that total correctness can be proved by establishing partial correctness (using denotational methods) and strong convergence (using operational methods). We illustrate the technique with a representative example.

Probability and Domain Theory
Michael MISLOVE (Tulane Univ.)

Gelfand Duality for Stably Compact Spaces
Andrew MOSHIER (Chapman Univ.)

The continuous, real valued functions over a compact Hausdorff space X form an ordered ring $C(X)$ with operations and order defined point-wise. The ordered rings that arise this way are characterized as those that admit a (unique) Q-algebra structure, and are bounded, archimedian and complete in the obvious uniformity derived from the Q-algebra structure. The classic result of Stone shows, in effect, that the category of such rings is dual to the category of compact Hausdorff spaces. In this talk, we extend this duality to stably compact spaces by considering the functor $P(-)$ taking a stably compact space to its semi-ring of perfect, non-negative, real valued functions. We characterize the semi-rings that arise as $P(X)$ and show that the category of these semi-rings is dually equivalent to the category of stably compact spaces and perfect functions. In addition, we show that the patch co-reflection in stably compact spaces translates to the construction of the free ring (simpliciter) over $P(X)$.

Points in quantales and *cm*-lattices
Jan PASENKA (Univ. Brno)

An important notion introduced in the context of irreducible representations of C^*-algebras is the notion of prime element of a quantale. In 2002, Kruml introduced the notion of a distributive quantale and proved that any algebraic distributive quantale is spatial.

Motivated by this result, by the results of the present author and by the work of Banaschewski and Erne in the context of two-sided quantales, we shall generalize the corresponding ideas concerning prime elements and semiprime ideals for arbitrary quantales and *cm*-lattices. In fact, it is precisely the lack of two-sidedness that causes some difficulties.

Topological methods in automata theory
Jean-Eric PIN (Univ. Paris VII)

Why Logics for Objects should have Denotational Semantics
Bernhard REUS and Jan SCHWINGHAMMER (Univ. Sussex)

Abadi-Leino logic is a Hoare-calculus style logic for a simple imperative and object-based language in which every object contains its own suite of methods. We present a new soundness proof for this logic using a denotational semantics where specifications are (recursive) predicates on a recursive domain (i.e. the store). Semantically, the existence of these specifications is not always guaranteed which provides an intuitive explanation of some of the inherent limitations of the programming logic and reveals where improvements are possible. Pitts' techniques are in use to deal with the existence problem. In our view, this proves the advantage of denotational semantics in soundness proofs of programming logics.

Concrete data structures as games
Andrea SCHALK (Univ. Manchester)

A result by Curien establishes that filiform concrete data structures can be viewed as games. We extend the idea to cover all stable concrete data structures. This necessitates a theory of games with an equivalence relation on positions. We present a faithful functor from the category of concrete data structures to this new category of games, allowing a game-like reading of the former. It is possible to restrict to a cartesian closed subcategory of these games, where the function space does not decompose and the product is given by the usual tensor product construction. There is a close connection between these games and graph games.

Flatness, preorders and general metric spaces

Vincent SCHMITT (Univ. Leicester)

This talk shall present a general notion of flatness in the enriched context: $\mathcal{P}$-flatness where the parameter $\mathcal{P}$ stands for a class of presheaves. One obtains a completion of a category A by considering the category $Flat_{\mathcal{P}}(A)$ of $\mathcal{P}$-flat presheaves over A. This completion is related to the free cocompletion under a class of colimits defined by Kelly. We define a notion of Q-accessible categories for a family Q of indices. Our $Flat_{\mathcal{P}}(A)$ for small A's are exactly the Q-accessible categories where Q is the class of $\mathcal{P}$-flat indices. For a category A, for $\mathcal{P} = \mathcal{P}_0$ the class of all presheaves, $Flat_{\mathcal{P}}(A)$ is the Cauchy completion of A. Two classes $\mathcal{P}_1$ and $\mathcal{P}_2$ of interest for general metric spaces are considered. The $\mathcal{P}_1$ and $\mathcal{P}_2$-flatness are investigated and the associated completions are characterized for general metric spaces (enrichments over $\bar{\mathbb{R}}^+$) and preorders (enrichments over **Bool**). We get this way two non-symmetric completions for metric spaces and retrieve the ideal completion for preorders.

Parametric Sets and Virtual Classes

Dana SCOTT (Carnegie Mellon University)

In an axiomatic development of geometry, there is much convenience to be found in treating various loci as sets. Thus, a line corresponds to the set of all points lying on the line; a circle, to the set of all points on the circumference. Moreover, sets of sets are natural, say in considering pencils of lines or circles or conics. And families of pencils are used as well. Does geometry need a full set theory, therefore? In giving a negative answer, we shall consider higher-type sets introduced by parametric definitions with just finite lists of points as parameters. An attempt will be made at axiomatizing such sets together with a notation for virtual classes. The objective is to have the use of set-theoretical notations without the ontology of higher-type logic and Zermelo-Fraenkel set theory.

Variations on Wadge Reducibility

Victor SELIVANOV (Siberian Division of the Russian Academy of Sciences)

Wadge reducibility in the Baire and Cantor spaces is very important in descriptive set theory. We consider Wadge reducibility in some other topological spaces, in particular in the φ-spaces which are topological counterparts of the algebraic directed-complete partial orderings.

We try to understand which properties (or their weaker versions) of the classical Wadge reducibility hold true in the other spaces. E.g., we discuss when some substructures of the Wadge ordering are almost well ordered, which sets have a supremum (or a weak version of supremum) under Wadge reducibility, and consider relationship of the Wadge reducibility to hierarchies considered by the author and by A. Tang.

We show that in many φ-spaces the behaviour of the Wadge reducibility resembles its behaviour in the Baire and Cantor spaces (at least within $\mathbf{\Delta}^0_2$). Moreover, we show that the study of Wadge reducibility in φ-spaces subsumes that in the Baire and Cantor space. In an earlier paper we observed that some results on Wadge reducibility in $P\omega$ have interesting implications for the theory of ω-ary boolean operations.

Hence, the study of Wadge reducibility in φ-spaces (especially in concrete important spaces like $P\omega$) seems to be a natural developement of the classical theory of Wadge degrees and deserves, in our opinion, further work.

Domains with Approximation Structure and Their Canonical Quasi-Metrics

Dieter SPREEN (Univ. Siegen)

Approximation structures have been introduced for a special class of Scott domains some years ago. They were used to construct new models for the untyped lambda calculus. In his PhD thesis Kummetz generalised this approach and showed that such spaces have a canonical uniformity. As is easy to see this uniformity can be obtained by symmetrising a quasi-uniformity.

The question is studied when the corresponding topology coincides with the Scott topology of the underlying domain. Moreover, implications of this result for the above mentioned domains with approximation structure are considered.

Tychonov's theorem in Abstract Stone Duality

Paul TAYLOR (Univ. Manchester)

Dimensional Properties of Domains

Hideki TSUIKI (Univ. Kyoto)

In this talk, we show that the weak inductive dimension of the set $L(D)$ of limit (i.e. non-compact) elements of an algebraic domain with property M (i.e. Lawson-compact) is equal to the height of $L(D)$.

As a consequence, a compact metric space X has dimension n iff it is a retract of $L(D)$ for an ω-algebraic domain D which has property M and whose set of limits $L(D)$ has height n.

Partial Horn Logic

Steve VICKERS (Univ. Birmingham)

Various logics for partially defined terms have been investigated in the past. I shall describe one based on logic in the form of sequents labelled by contexts à la Mostowski (and as presented in Johnstone's *Elephant* part D). The logic is virtually unchanged except for the substitution rule, which must incorporate a condition for definedness of terms.

If t is a term in context $\{y\}$ with sort compatible with $\{x\}$, then the rule is:

$$\frac{\phi \vdash_{\{x\}} \psi}{t = t \wedge \phi[t/x] \vdash_{\{y\}} \psi[t/x]}$$

The self-equation $t = t$ represents definedness of t

This slight change has a big impact on the usability of cartesian theories, those corresponding to the categorical structure of cartesian (finite limit) categories. They are equivalent to partial Horn theories and indeed can dispense with predicates. We call these "quasi-equational theories". They have sorts, function symbols (interpreted partially), and axioms $phi \vdash_{\{} x\}\psi$ where ϕ and ψ are finite conjunctions of equations.

Using the Initial Model Theorem one can then prove

1. The Free Model Theorem. Suppose $f : T_1 \to T_2$ is a morphism of quasi-equational theories. Then the reduct functor from partial models of T_2 to those of T_1 has a left adjoint.

2. Each quasi-equational theory T has a classifying (syntactic) category $\mathbb{C}_T$, a cartesian category with a generic model of T, such that models of T in $\mathbb{D}$ are equivalent to cartesian functors from $\mathbb{C}_T$ to $\mathbb{D}$. $\mathbb{C}_T$ is an initial cartesian-category-equipped-with-T-partial-model.

Examples of quasi-equational theories include those of categories, of categories with various kinds of structure including cartesian categories, of finitary sheaves over distributive lattices, and of conditional join semilattices - i.e. the compact bases of Scott domains.

Completeness and compactness of partialy metrized domains

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We will discuss the dependence between order-completeness and pmetric completeness (and compactness) of partially metrized domains.