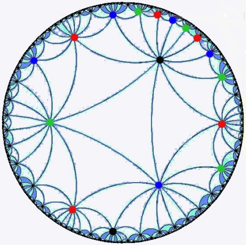
Groupoids, Hypergraphs, and Symmetries

Martin Otto

AIMoTh 13, Berlin



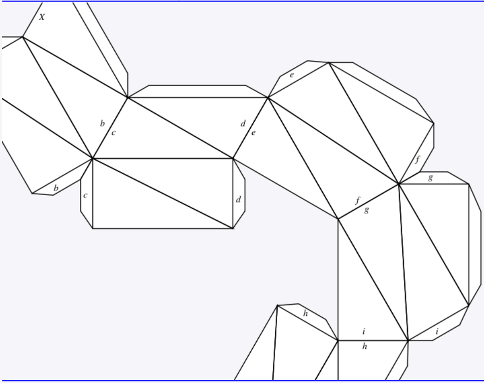
tesselation images created with http://aleph0.clarku.edu/ djoyce/poincare/PoincareApplet.html

combinatorial questions:

- **can every overlap pattern of hyperedges be realised in some finite hypergraph?**
- **does every finite hypergraph admit finite covers of any degree of acyclicity?**
- **how faithful can finite realisations/covers be w.r.t. symmetries of the specification?**

hypergraphs $\mathcal{A} = (A, S)$, $S \subseteq \mathcal{P}(A)$ occur as abstractions
of relational structures/data bases,
of cluster patterns of variables in CSP,
as combinatorial patterns of structural decompositions, ...

bastelbogen/glueing by numbers



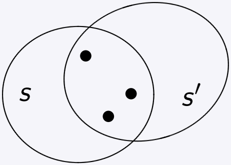
⇝ **here just combinatorics** (sets & local bijections)

specification: local overlap pattern
realisation: local consistency & finite closure

bastelbogen/glueing by numbers

concrete specification: as in given hypergraph

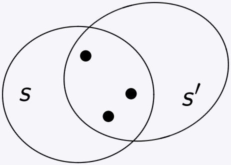
reproduce intersection pattern of given $\mathcal{A} = (A, S)$
$I(\mathcal{A}) = (S, E)$, $E = \{(s, s') \colon s \neq s', s \cap s' \neq \emptyset\}$



bastelbogen/glueing by numbers

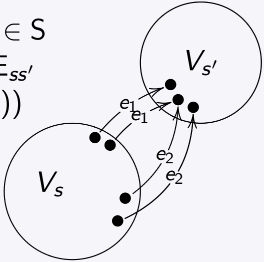
concrete specification: as in given hypergraph

reproduce intersection pattern of given $\mathcal{A} = (\mathsf{A}, \mathsf{S})$
$\mathsf{I}(\mathcal{A}) = (\mathsf{S}, \mathsf{E})$, $\mathsf{E} = \{(s, s') \colon s \neq s', s \cap s' \neq \emptyset\}$



abstract specification

of overlaps between disjoint patches V_s for $s \in \mathsf{S}$
via partial matchings $\mathsf{R}_e \subseteq \mathsf{V}_s \times \mathsf{V}_{s'}$ for $e \in \mathsf{E}_{ss'}$
according to intersection pattern $\mathsf{I} = (\mathsf{S}, (\mathsf{E}_{ss'}))$



. . . a valid template for a finite hypergraph?

overlap patterns & their realisations

bastelbogen: the abstraction

incidence pattern: sites and links

$\mathbf{I = (S, E)}$ multigraph, $\mathrm{E} = (\mathrm{E}_{ss'} \colon s, s' \in \mathrm{S})$

with involutive edge reversal $e \in \mathrm{E}_{ss'} \longmapsto e^{-1} \in \mathrm{E}_{s's}$

overlap patterns & their realisations

bastelbogen: the abstraction

incidence pattern: sites and links

$\mathbf{I = (S, E)}$ multigraph, $\mathrm{E} = (\mathrm{E}_{ss'} \colon s, s' \in \mathrm{S})$

with involutive edge reversal $e \in \mathrm{E}_{ss'} \longmapsto e^{-1} \in \mathrm{E}_{s's}$

I-graph: local overlap pattern

$\mathbf{H = (V, (V_s)_{s \in S}, (R_e)_{e \in E})}$

partitioned into patches V_s, connected by partial matchings $\mathrm{R}_e \subseteq \mathrm{V}_s \times \mathrm{V}_{s'}$ for $e \in \mathrm{E}_{ss'}$ compatible with $e \mapsto e^{-1}$

overlap patterns & their realisations

bastelbogen: the abstraction

incidence pattern: sites and links

$\mathbf{I} = (\mathbf{S}, \mathbf{E})$ multigraph, $\mathrm{E} = (\mathrm{E}_{ss'} \colon s, s' \in \mathrm{S})$

with involutive edge reversal $e \in \mathrm{E}_{ss'} \longmapsto e^{-1} \in \mathrm{E}_{s's}$

I-graph: local overlap pattern

$\mathbf{H} = (\mathbf{V}, (\mathbf{V_s})_{\mathbf{s} \in \mathbf{S}}, (\mathbf{R_e})_{\mathbf{e} \in \mathbf{E}})$

partitioned into patches V_s, connected by partial matchings $\mathrm{R}_e \subseteq \mathrm{V}_s \times \mathrm{V}_{s'}$ for $e \in \mathrm{E}_{ss'}$ compatible with $e \mapsto e^{-1}$

realisation: global realisation of local specification

$\hat{\mathcal{A}} = (\hat{\mathbf{A}}, \hat{\mathbf{S}})$ hypergraph, with projection $\pi \colon \hat{\mathrm{S}} \longrightarrow \mathrm{S}$ and local bijections $\pi_{\hat{s}} \colon \hat{s} \longrightarrow \mathrm{V}_{\pi(\hat{s})}$ realising matchings as overlaps

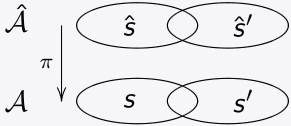
example 1: hypergraph coverings

bisimilar covering: $\boldsymbol{\pi : \hat{\mathcal{A}} \rightarrow \mathcal{A}}$

of hypergraph $\mathcal{A} = (\mathsf{A}, \mathsf{S})$ by hypergraph $\hat{\mathcal{A}} = (\hat{\mathsf{A}}, \hat{\mathsf{S}})$

(forth): π is a homomorphism mapping hyperedges $\hat{\mathsf{s}} \in \hat{\mathsf{S}}$ bijectively onto hyperedges $\pi(\hat{\mathsf{s}}) = \mathsf{s} \in \mathsf{S}$

(back): π lifts overlaps $\mathsf{s} \cap \mathsf{s}'$ to any $\hat{\mathsf{s}}$ above s:



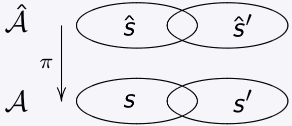
example 1: hypergraph coverings

bisimilar covering: $\boldsymbol{\pi} : \hat{\mathcal{A}} \rightarrow \mathcal{A}$

of hypergraph $\mathcal{A} = (\mathsf{A}, \mathsf{S})$ by hypergraph $\hat{\mathcal{A}} = (\hat{\mathsf{A}}, \hat{\mathsf{S}})$

(forth): π is a homomorphism mapping hyperedges $\hat{\mathsf{s}} \in \hat{\mathsf{S}}$ bijectively onto hyperedges $\pi(\hat{\mathsf{s}}) = \mathsf{s} \in \mathsf{S}$

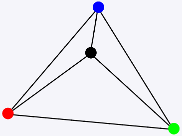
(back): π lifts overlaps $\mathsf{s} \cap \mathsf{s}'$ to any $\hat{\mathsf{s}}$ above s:



realisations of an overlap specification based on
$\mathbf{I}(\mathcal{A}) := (\mathbf{S}, \mathbf{E}),\ \ \mathbf{E} = \{(\mathbf{s}, \mathbf{s}') : \mathbf{s} \neq \mathbf{s}', \mathbf{s} \cap \mathbf{s}' \neq \emptyset\}$
and specification of $\mathcal{A}$ as I-graph

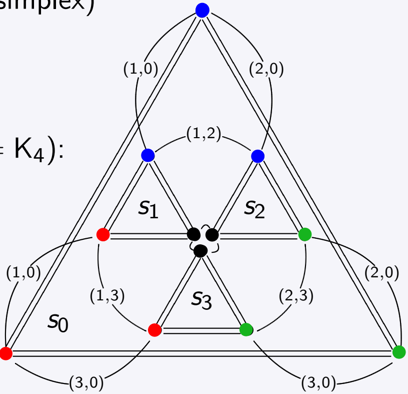
issues: degrees of acyclicity, saturation and symmetry

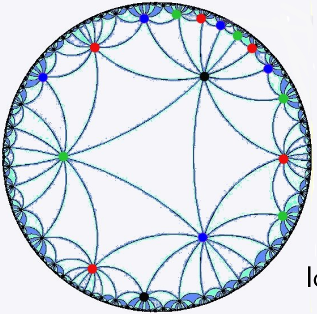
example



$[4]^3$ =, full 3-uniform hypergraph on 4 vertices (facets of 3-simplex)

I-graph representation ($I = K_4$):





locally finite (locally 4-fold) covering

example 2: realisation of GF-types

implicit specification of complete GF-theory

$\rightsquigarrow$ **abstract specification of overlap pattern of guarded tuples/substructures**

find models as realisations of overlap pattern that avoid local inconsistencies (by avoiding short cycles)

issues: acyclicity-, saturation- & symmetry properties

example 3: extension of partial isomorphisms

Hrushovski–Herwig–Lascar EPPA

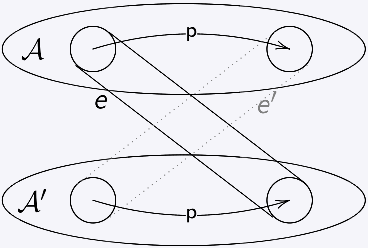
for finite relational structure $\mathcal{A} = (\mathsf{A}, \mathbf{R})$ and $\mathsf{p} \in \mathsf{Part}(\mathcal{A}, \mathcal{A})$
find *finite* extension $\mathcal{B} \supseteq \mathcal{A}$ such that p extends to $\check{\mathsf{p}} \in \mathsf{Aut}(\mathcal{B})$

example 3: extension of partial isomorphisms

Hrushovski–Herwig–Lascar EPPA

for finite relational structure $\mathcal{A} = (\mathsf{A}, \mathbf{R})$ and $\mathsf{p} \in \mathrm{Part}(\mathcal{A}, \mathcal{A})$
find *finite* extension $\mathcal{B} \supseteq \mathcal{A}$ such that p extends to $\check{\mathsf{p}} \in \mathrm{Aut}(\mathcal{B})$

use overlap pattern between copies $\mathcal{A}, \mathcal{A}'$ and glueing of $\mathrm{dom}(\mathsf{p}) \subseteq \mathsf{A}, \mathsf{A}'$ and $\mathrm{image}(\mathsf{p}) \subseteq \mathsf{A}', \mathsf{A}$



symmetry & acyclicity properties at stake!

in this talk

- graph/hypergraph coverings
- (reduced) products with groups/groupoids
- degrees of acyclicity in products with groups/groupoids without short coset cycles
- genericity and symmetries of these constructions

a remark on symmetries

symmetries of coverings $\pi \colon \hat{\mathcal{A}} \longrightarrow \mathcal{A}$ are of two kinds

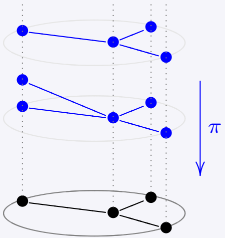
- **vertical:** fibre-preserving symmetries relating $\hat{s}_1, \hat{s}_2 \in \pi^{-1}(s)$.

- **horizontal:** lifting symmetries of base/specification

natural/generic coverings should provide both!

graph covering $\pi \colon \hat{\mathcal{A}} \to \mathcal{A}$

of graph $\mathcal{A} = (\mathsf{A}, \mathsf{E})$ by graph $\hat{\mathcal{A}} = (\hat{\mathsf{A}}, \hat{\mathsf{E}})$

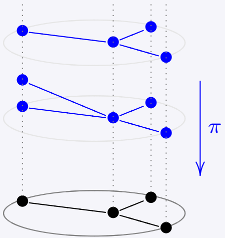


(forth): π homomorphically maps edges $\hat{\mathsf{e}} \in \hat{\mathsf{E}}$ onto edges $\pi(\hat{\mathsf{e}}) = \mathsf{e} \in \mathsf{E}$

(back): π lifts every edge e incident at $a \in A$ to any $\hat{a} \in \pi^{-1}(a)$ above a

graph covering $\pi\colon \hat{\mathcal{A}} \to \mathcal{A}$

of graph $\mathcal{A} = (\mathrm{A}, \mathrm{E})$ by graph $\hat{\mathcal{A}} = (\hat{\mathrm{A}}, \hat{\mathrm{E}})$



(forth): π homomorphically maps edges $\hat{\mathrm{e}} \in \hat{\mathrm{E}}$ onto edges $\pi(\hat{\mathrm{e}}) = \mathrm{e} \in \mathrm{E}$

(back): π lifts every edge e incident at $a \in A$ to any $\hat{a} \in \pi^{-1}(a)$ above a

thm O 2002

for finite graph $\mathcal{A} = (\mathrm{A}, \mathrm{E})$ and $\mathrm{N} \in \mathbb{N}$
find finite coverings $\pi\colon \hat{\mathcal{A}} \longrightarrow \mathcal{A}$ such that

- $\hat{\mathcal{A}}$ is N-acyclic **(no cycles up to length N)**
- π preserves incidence degrees **(an unbranched covering)**
- full symmetry is achieved **(generic product construction)**

Cayley groups of large girth

Cayley group & graph:

group $\mathbb{G} = \mathbb{G}[\mathsf{E}] = (\mathsf{G}, \cdot, 1)$ with finite set E of generators $e \in \mathsf{E}$

E-edge-coloured graph with edge relations
$\mathsf{R}_e = \{(g, g \cdot e) \colon g \in \mathsf{G}\}$ for $e \in \mathsf{E}$

girth: minimal length of generator cycle $e_1 \cdots e_n = 1$

Cayley groups of large girth

Cayley group & graph:

group $\mathbb{G} = \mathbb{G}[\mathsf{E}] = (\mathsf{G}, \cdot, 1)$ with finite set E of generators $e \in \mathsf{E}$

E-edge-coloured graph with edge relations
$\mathsf{R}_e = \{(g, g \cdot e) \colon g \in \mathsf{G}\}$ for $e \in \mathsf{E}$

girth: minimal length of generator cycle $e_1 \cdots e_n = 1$

generic construction idea (Alon–Biggs)**:**

obtain $\mathbb{G}$ as $\langle \pi_e \colon e \in \mathsf{E} \rangle \subseteq \mathrm{Sym}(\mathsf{V})$ for permutations π_e
on E-edge-coloured graph $\mathsf{H} = (\mathsf{V}, (\mathsf{R}_e)_{e \in \mathsf{E}})$

Cayley groups of large girth

Cayley group & graph:

group $\mathbb{G} = \mathbb{G}[\mathsf{E}] = (\mathsf{G}, \cdot, 1)$ with finite set E of generators $e \in \mathsf{E}$

E-edge-coloured graph with edge relations
$\mathsf{R}_e = \{(g, g \cdot e) : g \in \mathsf{G}\}$ for $e \in \mathsf{E}$

girth: minimal length of generator cycle $e_1 \cdots e_n = 1$

generic construction idea (Alon–Biggs)**:**

obtain $\mathbb{G}$ as $\langle \pi_e : e \in \mathsf{E} \rangle \subseteq \mathrm{Sym}(\mathsf{V})$ for permutations π_e
on E-edge-coloured graph $\mathsf{H} = (\mathsf{V}, (\mathsf{R}_e)_{e \in \mathsf{E}})$

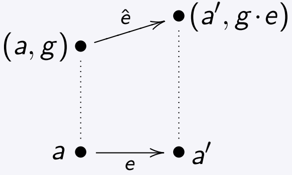
boosting girth (minimal sequence of π_e generating id_V)**:**

eliminate short cycles through embedded acyclic traces in H

direct (synchronous) products

between $\mathcal{A} = (\mathsf{A}, \mathsf{E})$ and Cayley groups $\mathbb{G}[\mathsf{E}]$

$$\mathcal{A} \otimes \mathbb{G} = (\hat{\mathsf{A}}, \hat{\mathsf{E}}) =: \begin{cases} \hat{\mathsf{A}} = \mathsf{A} \times \mathsf{G} \\ \hat{\mathsf{E}} \text{ lifting } \; e = (a, a') \in \mathsf{E} \\ \text{to } ((a, g), (a', g \cdot e)) \in \hat{\mathsf{E}} \end{cases}$$



provide unbranched coverings $\boldsymbol{\pi : \mathcal{A} \otimes \mathbb{G} \longrightarrow \mathcal{A}}$

- **N-acyclic if** $\mathrm{girth}(\mathbb{G}) > \mathsf{N}$
- **with all desirable symmetries for the right** $\mathbb{G}$

more than large girth: no short coset cycles

and one step towards hyperedeges

thm O 2010

obtain Cayley groups that are N-acyclic in the stronger sense of not admitting *coset cycles* of length up to N

more than large girth: no short coset cycles

and one step towards hyperedeges

thm O 2010

obtain Cayley groups that are N-acyclic in the stronger sense of not admitting *coset cycles* of length up to N

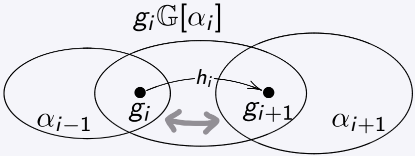
coset cycles:

in $\mathbb{G} = \mathbb{G}[\mathsf{E}]$ consider subgroups $\mathbb{G}[\alpha]$ generated by $\alpha \subseteq \mathsf{E}$

coset g$\mathbb{G}[\alpha]$ ⇜⇝ α-component of g

$(g_i, \alpha_i)_{i \in \mathbb{Z}_n}$ forms a coset cycle if

- $h_i := g_i^{-1} \cdot g_{i+1} \in \mathbb{G}[\alpha_i]$
- $g_i \mathbb{G}[\alpha_i \cap \alpha_{i-1}] \cap g_{i+1} \mathbb{G}[\alpha_i \cap \alpha_{i+1}] = \emptyset$

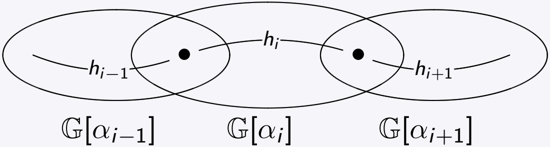


Cayley groups without short coset cycles

method: Biggs & amalgamation

inductively eliminate short coset cycles in $\mathbb{G}[\alpha]$ for larger α

induction step: $\mathbb{G} \rightsquigarrow \mathbb{G}' := \langle \pi_e \colon e \in \mathsf{E} \rangle \subseteq \mathrm{Sym}(\mathsf{V})$
in E-edge coloured graph containing
amalgamated chains of small cosets
'small coset cycles' unfolded



useful in hypergraph transformations (JACM 2012)
but not a stand-alone method for covers

finally to hypergraphs

with hypergraph $\mathcal{A} = (\mathsf{A}, \mathsf{S})$ associate

incidence pattern: $\mathsf{I}(\mathcal{A}) = (\mathsf{S}, \mathsf{E})$
$\mathsf{E} = \{(s, s') \colon s \neq s', s \cap s' \neq \emptyset\}$

Gaifman graph: $\mathsf{G}(\mathcal{A}) = (\mathsf{A}, \mathsf{R})$
$\mathsf{R} = \{(a, a') \colon a \neq a';\ a, a' \in s \text{ for some } s \in \mathsf{S}\}$

hyperedges $\rightsquigarrow$ cliques

finally to hypergraphs

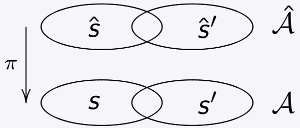
with hypergraph $\mathcal{A} = (\mathsf{A}, \mathsf{S})$ associate

incidence pattern: $\mathsf{I}(\mathcal{A}) = (\mathsf{S}, \mathsf{E})$
$\mathsf{E} = \{(s, s') \colon s \neq s', s \cap s' \neq \emptyset\}$

Gaifman graph: $\mathsf{G}(\mathcal{A}) = (\mathsf{A}, \mathsf{R})$
$\mathsf{R} = \{(a, a') \colon a \neq a';\ a, a' \in s \text{ for some } s \in \mathsf{S}\}$

hyperedges $\rightsquigarrow$ cliques

recall that a covering $\pi \colon \hat{\mathcal{A}} \to \mathcal{A}$ is a homomorphism
with *back*-property w.r.t. overlaps (lifting)



issue: degrees of acyclicity

hypergraph acyclicity

equivalent for finite $\mathcal{A} = (\mathsf{A}, \mathsf{S})$:

- $\mathcal{A}$ has tree decomposition with bag set S
- $\mathcal{A}$ dissolvable through retractions
- **$\mathcal{A}$ conformal & chordal**

conformal: every clique in $\mathsf{G}(\mathcal{A})$ induced by individual $s \in \mathsf{S}$

chordal: every cycle in $\mathsf{G}(\mathcal{A})$ of length > 3 has a chord

approximation: N-acyclicity (N-conformality & N-chordality)

guarantees acyclicity of induced sub-hypergraphs of size $\leqslant N$

hypergraph acyclicity

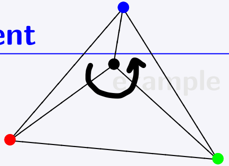
equivalent for finite $\mathcal{A} = (\mathsf{A}, \mathsf{S})$:

- $\mathcal{A}$ has tree decomposition with bag set S
- $\mathcal{A}$ dissolvable through retractions
- **$\mathcal{A}$ conformal & chordal**

conformal: every clique in $\mathsf{G}(\mathcal{A})$ induced by individual $s \in \mathsf{S}$

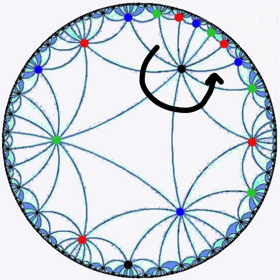
chordal: every cycle in $\mathsf{G}(\mathcal{A})$ of length > 3 has a chord

hypergraph coverings must be different



- non-trivial covers cannot preserve incidence degrees

  $\longrightarrow$ **no unbranched coverings**



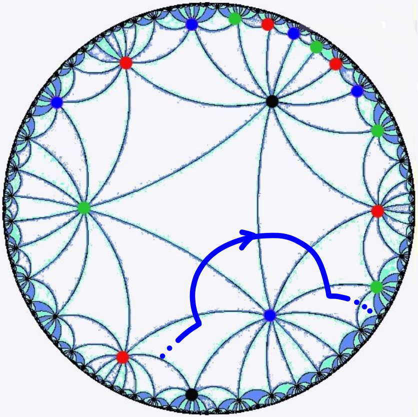
- non-trivial locality phenomena

  $\longrightarrow$ **need to control coset cycles**

- acyclicity not robust under passage to weak substructures

  $\longrightarrow$ **need to look to groupoids rather than groups**

why coset cycles may matter



why groupoids are more appropriate

Cayley graphs of groups $\mathbb{G}[\mathsf{E}]$ may be too homogeneous:
every generator available at every point
rather than just the necessary transitions/extensions

incidence pattern $\mathsf{I} = (\mathsf{S}, \mathsf{E}) = (\mathsf{S}, (\mathsf{E}_{ss'}))$
specifies which transitions are required locally

I-groupoid $\mathbb{G} = (\mathsf{G}, (\mathsf{G}_{st}), \cdot, (\lambda_s), {}^{-1})$

has sort-sensitive groupoidal operation $\mathsf{G}_{st} \times \mathsf{G}_{tu} \longrightarrow \mathsf{G}_{su}$,
neutral elements λ_s, and inverses $g^{-1} \in \mathsf{G}_{ts}$ for $g \in \mathsf{G}_{st}$

$\mathbb{G}$**[E]:** generated by groupoid elements $e \in \mathsf{G}_{ss'}$ for $e \in \mathsf{E}_{ss'}$

$\rightsquigarrow$ **homomorphism of I-paths** $s \xrightarrow{e_1} \cdots \xrightarrow{e_k} t$
onto groupoid elements $e_1 \cdots e_k \in \mathsf{G}_{st}$

!

N-acyclic Cayley groupoids

. . . inductive process for generating I-groupoids without short coset cycles from amalgamated chains yields

thm O 2012

for any $I = (S, (E_{ss'}))$ and for any $N \in \mathbb{N}$, there are finite I-groupoids $\mathbb{G}[E]$ without coset cycles of length up to N

can serve as generic factors for construction of hypergraphs with specified overlap pattern !

reduced products $\mathcal{A} \otimes \mathbb{G}$, $\mathbf{H} \otimes \mathbb{G}$, . . .

simple idea:

$$\left.\begin{array}{l}\text{start from disjoint union of copies of} \\ \text{hyperedges associated with } s \in \mathrm{S} \\ \text{indexed by elements } g \in \mathrm{G}_{*s} \subseteq \mathbb{G}\end{array}\right\} \quad \text{a direct product}$$

reduce to quotient w.r.t. equivalence $\approx$
induced by local identifications R_e/ρ_e

reduced products $\mathcal{A} \otimes \mathbb{G}$, $\mathbf{H} \otimes \mathbb{G}$, ...

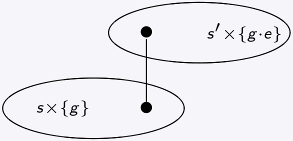
simple idea:

start from disjoint union of copies of hyperedges associated with $s \in \mathrm{S}$ indexed by elements $g \in \mathrm{G}_{*s} \subseteq \mathbb{G}$ } a direct product

reduce to quotient w.r.t. equivalence $\approx$ induced by local identifications R_e/ρ_e

e.g., for $e = (s, s') \in E$ in $\mathrm{I}(\mathcal{A})$:

$(a, g) \;\approx\; (a, g \cdot e)$ for $a \in s \cap s'$



reduced products $\mathcal{A} \otimes \mathbb{G}$, $\mathbf{H} \otimes \mathbb{G}$, ...

simple idea:

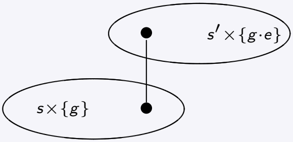
start from disjoint union of copies of
hyperedges associated with $s \in \mathrm{S}$
indexed by elements $g \in \mathrm{G}_{*s} \subseteq \mathbb{G}$

} a direct product

reduce to quotient w.r.t. equivalence $\approx$
induced by local identifications R_e/ρ_e

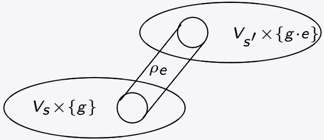
e.g., for $e = (s, s') \in E$ in $\mathrm{I}(\mathcal{A})$:

$(a, g) \;\approx\; (a, g\cdot e)$ for $a \in s \cap s'$



e.g., for $e \in \mathrm{E}_{ss'}$
and R_e/ρ_e of $\mathrm{H} = (\mathrm{V}, (\mathrm{V}_s), (\mathrm{R}_e))$:

$(v, g) \;\approx\; (v', g\cdot e)$ if $\rho_e(v) = v'$



acyclicity in reduced products

lemma (e.g. for coverings)

for hypergraph $\mathcal{A}$ and I-groupoid $\mathbb{G}$, where $\mathsf{I} = \mathsf{I}(\mathcal{A})$:

- $\pi\colon \mathcal{A} \otimes \mathbb{G} \longrightarrow \mathcal{A}$ is a covering;
- if $\mathbb{G}$ has no coset cycles of length up to N, then $\mathcal{A} \otimes \mathbb{G}$ is N-acyclic
- by construction, $\mathcal{A} \otimes \mathbb{G}$ has full vertical symmetry
- if $\mathbb{G}$ is I-symmetric, then $\mathcal{A} \otimes \mathbb{G}$ lifts all symmetries of $\mathcal{A}$

acyclicity in reduced products

lemma (e.g. for coverings)

for hypergraph $\mathcal{A}$ and I-groupoid $\mathbb{G}$, where $\mathsf{I} = \mathsf{I}(\mathcal{A})$:

- $\pi \colon \mathcal{A} \otimes \mathbb{G} \longrightarrow \mathcal{A}$ is a covering;
- if $\mathbb{G}$ has no coset cycles of length up to N, then $\mathcal{A} \otimes \mathbb{G}$ is N-acyclic
- by construction, $\mathcal{A} \otimes \mathbb{G}$ has full vertical symmetry
- if $\mathbb{G}$ is I-symmetric, then $\mathcal{A} \otimes \mathbb{G}$ lifts all symmetries of $\mathcal{A}$

(a, g) is identified, via $\approx$, precisely with (a, g')
in layers $g' \in g\mathbb{G}[\alpha_a]$, where $\alpha_a = \{(s, s') \colon a \in s \cap s'\}$

— **this is why coset cycles matter for acyclicity**

further applications of these reduced products

. . . finite symmetric realisations for specifications of overlap patterns and GF-types, and a new route to

Herwig–Lascar EPPA

construct finite extensions of finite relational $\mathcal{A}$ that extend specified partial isomorphisms of $\mathcal{A}$ to full automorphisms

strengthening: fmp within class $\mathcal{C}$ with finitely many forbidden homomorphisms

further applications of these reduced products

. . . finite symmetric realisations for specifications of overlap patterns and GF-types, and a new route to

Herwig–Lascar EPPA

construct finite extensions of finite relational $\mathcal{A}$ that extend specified partial isomorphisms of $\mathcal{A}$ to full automorphisms

strengthening: fmp within class $\mathcal{C}$ with finitely many forbidden homomorphisms

obtain hypergraph with copies of $\mathcal{A}$ as hyperedges

- realisation of local identifications between copies of $\mathcal{A}$
- desired automorphisms arise as symmetries w.r.t. specification