?????????Questions¿¿¿¿¿¿¿¿

Inquisitive Epistemic Modal Logic, teams, bisimulation and all that

AIMoTh
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Martin Otto joint work with Ivano Ciardelli

content

- **inquisitve modal & epistemic logic IML**
  a team semantic level on top of modal logic

- **inquisitive bisimulation & Ehrenfeucht-Fraïssé**
  back&forth somewhere between FO and MSO

- **characterisation theorems IML $\equiv$ FO$/\!\sim$**
  over suitable two-sorted relational structures

modal structures

Kripke structures $\mathcal{K} = (W, R, \rho)$
for the semantics of basic modal logic ML

- set of possible worlds W
- propositional assignment $\rho\colon p \mapsto \rho(p) \in 2^W$
  for semantics of proposition p in each world w
- accessibility relation(s) $R \subseteq W \times W$
  for semantics of modal $\Box/\Diamond$

  or the function

  $$\sigma\colon u \mapsto \sigma(u) := R[u] = \{v \in W\colon (u, v) \in R\} \in 2^W$$

  set of accessible worlds:
  information state at u

from modal to inquisitive

- modal assignment of sets of accessible worlds:

  $\sigma \colon u \quad \longmapsto \quad \sigma(u) = R[u] \in 2^W$ the *information state* at u

  semantics of modal $\Box/\Diamond$ at u: FO-quantification over $\sigma(u)$

  in epistemic reading: $\sigma(u) =$ (lack of) knowledge in u

  information state at u

  a set of possible worlds at u

to give semantics to questions add

- inquisitive assignment of sets of "information updates":

  $\Sigma \colon u \quad \longmapsto \quad \Sigma(u) \in 2^{2^W}$ the *inquisitive state* at u

  in epistemic reading: $\Sigma(u) =$ possible updates in u

  inquisitive state at u

  a set of information states at u

inquisitive structures (functional format)

augment Kripke structures $\mathcal{K} = (W, \sigma, \rho)$
for the semantics of basic modal logic ML
to inquisitive structures $\mathcal{K} = (W, \Sigma, \rho)$

- set of possible worlds W
- propositional assignment $\rho\colon p \mapsto \rho(p) \in \rho(p) \in 2^W$
  for semantics of proposition p in each world w
- inquisitive assignment(s) $\Sigma\colon u \longmapsto \Sigma(u) \in 2^{2^W}$
  for semantics of new inquisitive modalities

with induced

- modal assignment(s) $\sigma\colon u \longmapsto \bigcup \Sigma(u) \in 2^W$
  for semantics of modal $\Box/\Diamond$

inquisitive structures (relational format)

from Kripke structures $\mathcal{K} = (W, R, \rho)$ with $R \subseteq W \times W$
to inquisitive structures $\mathbb{K} = (W, E, \rho)$ with $E \subseteq W \times 2^W$

encode $\Sigma \colon u \mapsto \Sigma(u) \in 2^{2^W}$ by its graph $E \subseteq W \times 2^W$
in a two-sorted relational structure with

- first sort: possible worlds, W
- second sort: information states, $S \subseteq 2^W$

linked by two mixed-sorted relations in $W \times S$:

- set-theoretic $\in \subseteq W \times S$ (built-in like $=$)
- $E \subseteq W \times S$ (the graph of Σ)

with induced modal accessibility relation(s)

- $R = E \circ \in^{-1}$ (the graph of $\sigma \colon u \mapsto \bigcup \Sigma(u)$)

inquisitive modal logic IML $\supseteq$ ML

satisfaction relation, team semantic style,

between information states over $\mathbb{K}$, i.e. $s \in 2^W$ and formulae $\varphi \in \mathrm{IML}$

read $\mathbb{K}, s \models \varphi$ as "s supports φ"

with $\mathbb{K}, \{w\} \models \varphi$ emulating $\mathbb{K}, w \models \varphi$ for $\varphi \in \mathrm{ML}$

semantic constraints on models:

- inquisitive assignments $\Sigma(u)$ downward closed in 2^W **!**

and for (multi-agent) epistemic setting:

- induced modal σ_a/R_a are (S5) with classes $[u]_a = \sigma_a(u)$
- each Σ_a constant on its equivalence classes $[u]_a = \sigma_a(u)$

syntax and semantics for IML $\supseteq$ ML

atoms $p, \bot$: $\mathbb{K}, s \models p$ if $s \subseteq \rho(p)$ **flat**

$\mathbb{K}, s \models \bot$ iff $s = \emptyset$

strong disjunction $\veebar$: $\mathbb{K}, s \models \varphi_1 \veebar \varphi_2$ if $\mathbb{K}, s \models \varphi_1$ or $\mathbb{K}, s \models \varphi_2$ **non-flat**

team implication $\rightarrow$: $\mathbb{K}, s \models \varphi \rightarrow \psi$ if for all $s' \subseteq s$ $\mathbb{K}, s' \models \varphi \;\Rightarrow\; \mathbb{K}, s' \models \psi$ **non-flat**

inquisitive modalities $\boxplus$: **flattening**

$$\mathbb{K}, s \models \boxplus\varphi \text{ if } \mathbb{K}, s' \models \varphi \text{ for all } s' \in \Sigma(w), w \in s$$

induced plain modalities $\Box$: **flattening**

$$\mathbb{K}, s \models \Box\varphi \text{ if } \mathbb{K}, \{v\} \models \varphi \text{ for all } v \in \sigma(w), w \in s$$

some examples (involving questions)

$?\varphi := \varphi \veebar \neg\varphi$ captures "question *whether* φ is settled"
irrespective of "which way"

crucially non-flat

	supported by s in $\mathbb{K}$ iff
$?\varphi$	"s settles φ"
$\boxplus\,?\,\varphi$	"every information update in s will settle φ"
$\Box\,?\,\varphi$	"all information updates in s settle φ the same"
$\neg\Box\,?\,\varphi \wedge \boxplus\,?\,\varphi$	"the open question φ will be answered in s"

inquisitive bisimulation and Ehrenfeucht–Fraïssé

back&forth game on $\mathbb{K}$ and $\mathbb{K}'$ with split rounds

from (u, u')	from matching worlds
to $(s, s') \in \Sigma(u) \times \Sigma'(u')$	via information states
to $(w, w') \in s \times s'$	to matching worlds

$\rightsquigarrow$ natural notions of bisimilarity $\sim$ / $\sim^\ell$

$\rightsquigarrow$ Ehrenfeucht–Fraïssé link between $\sim^\ell$ and $\equiv_\ell^{\mathsf{IML}}$
with non-trivial characteristic formulae at world/state levels

in particular **IML is $\sim$-invariant**

bisimulation invariance & compactness

in relational format, the actual extension of the second sort $S \subseteq 2^W$ in $\mathbb{K} = (W, S, E, \rho)$ is relatively free up to $\sim$

$$\text{natural levels: } \begin{cases} S = 2^W & \text{full/maximal} \\ S \supseteq \bigcup_{u \in W} 2^{\sigma(u)} & \text{locally full} \\ S \supseteq \bigcup_{s \in \Sigma(u)} 2^s & \text{min. req.} \end{cases}$$

for downward closed S **IML** $\subseteq$ **FO**/$\sim$
and **IML** $\supseteq$ **FO**/$\sim$ is equivalent to

$\sim$-invariance $\Rightarrow$ $\sim^\ell$-invariance for some $\ell \in \mathbb{N}$

$\rightsquigarrow$ upgrading constructions

limitation: over full/maximal format, FO/$\sim$ fails to satisfy compactness, whence IML $\not\equiv$ FO/$\sim$

IML is known to be compact

characterisation theorems

$\rightsquigarrow$ expressive completeness for van Benthem–Rosen style characterisations of IML:

$$\mathrm{FO}/\!\sim\; \equiv \mathrm{IML} \text{ over } \mathcal{C} \quad \text{(classically and fmt)}$$

over the above feasible classes of two-sorted relational inquisitive structures

> all these classes are non-elementary and combine FO and MSO features

through upgrading for compactness property collapsing $\sim$ to $\sim^\ell$

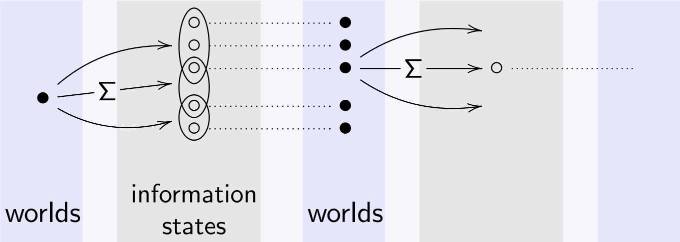
$$\begin{array}{ccc}
\mathbb{K}, w & \text{---}\sim^\ell\text{---} & \mathbb{K}', w' \\
\downarrow \sim & & \downarrow \sim \\
\hat{\mathbb{K}}, w & \text{---}\equiv_q\text{---} & \hat{\mathbb{K}}', w'
\end{array}$$

expressive completeness via upgrading (I)

towards FO/$\sim$ $\subseteq$ IML   e.g. over the classes $\mathcal{C}/\mathcal{C}_{\mathrm{fin}}$ of locally full relational inquisitive models

- upgrading $\sim^{\ell}$ to $\equiv_q$ over $\mathcal{C}/\mathcal{C}_{\mathrm{fin}}$ using FO-locality:

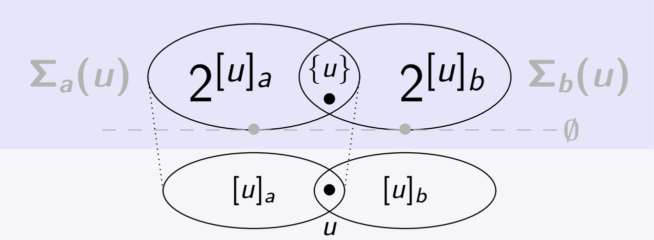
  local unfolding & world/state-layer stratification
  with fresh worlds to instantiate information states



expressive completeness via upgrading (II)

towards FO/$\sim$ $\subseteq$ IML e.g. over the classes $\mathcal{C}/\mathcal{C}_{\rm fin}$ of locally full relational inquisitive (S5) models

- local pre-processing of inquisitive assigments $\Sigma_a(u)$ in $[u]_a$
  $\rightsquigarrow$ simple lattice algebra & compositionality for unary MSO
- global pre-processing of overlap pattern between classes $[u]_a$
  $\rightsquigarrow$ treatment of (S5) Kripke structures in Dawar–O_09



what makes this interesting ...

- two-sortedness in a team semantic spirit
- non-trivial but tame intermediate level between FO and MSO
- another case of locality analysis near the limit (?)
  cf. work with Felix Canavoi on ML[CK] in LICS 17

$\longrightarrow$ Ciardelli–O_: TARK 17, results for basic IML,
& draft journal paper for (S5)

The End