Pinning and wetting transition for Δ - interaction

joint with F CARAVENNA

$$\varphi : \mathbb{Z} \longrightarrow \mathbb{R} \qquad \varphi_{-1} = \varphi_0 = 0 , \quad \varphi_N = \varphi_{N+1} = 0$$

$$\mathbb{P}_N^{\varepsilon}(d\varphi) = \frac{1}{Z_N(\varepsilon)} \exp(-H_N(\varphi)) \prod_{n=1}^{N-1} \left(d\varphi_n + \varepsilon\, \delta_0 (d\varphi_n) \right)$$

$$H_N(\varphi) = \sum_{n=0}^{N} V(\Delta \varphi(n)) \qquad \Delta \varphi(n) = \varphi(n+1) + \varphi(n-1) - 2\varphi(n)$$

$$V : \mathbb{R} \to \mathbb{R} \qquad \int e^{-V(y)} dy = 1 , \quad \int y e^{-V(y)} dy = 0$$

$$\int y^2 e^{-V(y)} dy = 1$$

$$\Omega_N^+ = \{ \varphi_n \geq 0 \quad 0 \leq n \leq N \} \qquad \mathbb{P}_N^{\varepsilon,+} = \mathbb{P}_N^{\varepsilon}(\quad | \Omega_N^+)$$

$\varepsilon = 0$ free model

$(Y_n)_{n \in \mathbb{N}}$ iid $\quad Y_n \sim e^{-V(y)} dy$

$$S_n = Y_1 + Y_2 + \quad + Y_n$$

$$A_n = S_1 + S_2 + \quad + S_n$$

$$\mathbb{P}_N^0 = \mathbb{P}\left((A_n)_{n=1}^N \;\middle|\; A_N = 0 ,\, A_{N+1} = 0 \right)$$

$$\mathbb{P}_N^{0,+} = \mathbb{P}\left((A_n)_{n=1}^N \;\middle|\; A_N = 0 ,\, A_{N+1} = 0 ,\, \Omega_N^+(A) \right)$$

<u>Invariance principle</u>

$$\mathbb{P}_N^0 \left(\left(\frac{1}{N^{3/2}} A_{\lfloor Nt \rfloor} \right)_{0 \le t \le 1} \in \cdot \right) \Longrightarrow \mathbb{P}\left(I(t) = \int_0^t \beta_s ds \in \cdot \;\middle|\; I(1), \beta(1) = 0 \right)$$

Free energy

$$F^{(+)}(\varepsilon) = \lim_{N \to \infty} \frac{1}{N} \log Z_N^{(1)}(\varepsilon), \qquad F^{(+)}(0) = 0$$

Localized phase $= \{ \varepsilon : F^{(+)}(\varepsilon) > 0 \} = (\varepsilon_c^{(+)}, \infty)$

THEOREM 1 $\exists \quad 0 < \varepsilon_c < \varepsilon_c^+ < \infty$,

$F^{(+)}$ is analytic on $(\bar{\varepsilon}_c^{(+)}, \infty)$

At ε_c^+ : first order transition, localized

At ε_c second order transition, delocalized

Let $\mathcal{T} = \{ n \in \mathbb{Z} : \varphi_n = 0 \}$ <u>Contact set</u>

$\ell_N = \# \mathcal{T} \cap [0, N]$ number of contacts

$\varepsilon > \varepsilon_c^{(+)}$ $\quad \mathbb{P}_N^{\varepsilon,(+)}\left(\left| \frac{\ell_N}{N} - D^{(+)}(\varepsilon) \right| > \delta \right) \leq e^{-C_1 N}$

$$D^{(+)}(\varepsilon) = \lim_{N \to \infty} \mathbb{E}_N^{\varepsilon,(+)}\left[\frac{\ell_N}{N} \right] > 0 \quad (> 0 \text{ for } \varepsilon = \varepsilon_c^+)$$

$\varepsilon < \varepsilon_c^{(+)}$ $\quad \mathbb{P}_N^{\varepsilon,(+)}\left(\frac{\ell_N}{N} > \delta \right) \leq e^{-C_2 N}$

$$\Delta_N = \max_{\ell < N} \{ \varphi_{n+1} \neq 0, \varphi_{n+2} \neq 0, \dots \varphi_{n+\ell} \neq 0 \quad n \leq N - \ell \}$$

<u>Maximal gap</u>

$$\lim_{N \to \infty} \mathbb{P}_N^{\varepsilon,(+)}\left(\frac{\Delta_N}{N} \geq \delta \right) = 0 \quad \varepsilon > \varepsilon_c^+$$

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$$\lim_{L\to\infty} \overline{\lim_{N\to\infty}} \mathbb{P}_N^{\varepsilon,(+)} \left(\Delta_N \le N - L \right) = 0 \qquad \varepsilon < \varepsilon_c^{(+)}$$

THEOREM 2 $\quad \varepsilon < \varepsilon_c$

$$\mathbb{P}_N^{\varepsilon}\left(\left(\frac{1}{N^{3/2}} \varphi_{[Nt]} \right)_{0\le t\le 1} \in \cdot \right) \Rightarrow \mathbb{P}\left(\left(I_t = \int_0^t \beta_s \, ds \right)_{0\le t\le 1} \,\Big|\, I_1 = 0, \beta_1 = 0 \right)$$

Open question for wetting model

$\varepsilon < \varepsilon_c^+$

$$\mathbb{P}_N^{\varepsilon,+}\left(\left(\frac{1}{N^{3/2}} \varphi_{[Nt]} \right)_{0\le t\le 1} \in \cdot \right) \overset{?}{\Rightarrow} \mathbb{P}^+\left(\left(I_t \right)_{t\le 1} \,\Big|\, I_1 = 0, \beta_1 = 0 \right)$$

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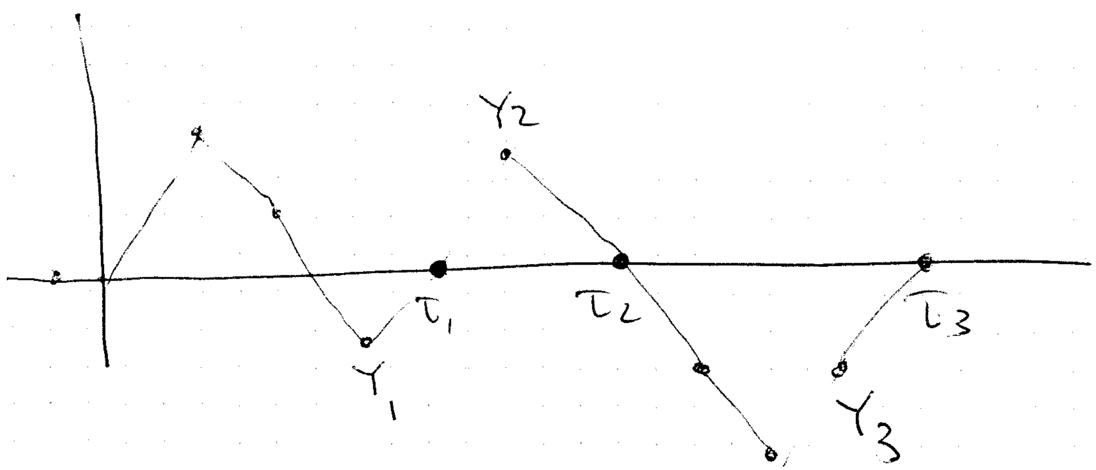
Idea of Proof: let $\tau \cap [0,N] = \{0, \tau_1, \dots, \tau_{\ell_{N}+1}\}$

$\tau_1 < \tau_2 < \dots < \tau_\ell$

be the contact set (i.e. $\varphi_{\tau_j} = 0$)

$Y_j = \varphi_{\tau_j - 1}$



$$\mathbb{P}_N^{\varepsilon,(+)}\left(\ell_N = \ell,\ \tau_i = t_i,\ J_i \in dy_i\ \ i = 1, \dots, \ell\right)$$

$$= \frac{1}{Z^{(+)}_{\varepsilon,N}}\, \varepsilon^{\ell-1} F^{(+)}_{0,dy_1}(t_1) \times F^{(+)}_{Y_1,dy_2}(t_2 - t_1) \cdots F^{(+)}_{Y_\ell,0}(1)$$

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$$F_{x,dy}(n) = f_{x,y}(n)\, dy$$

$f_{x,y}(n)$: density of $(\varphi_{n-1}, \varphi_n)$ at $(y,0)$ under $\mathbb{P}_{-x,0}$

$$F^+_{x,dy}(n) = \omega^+_{x,y}(n)\, f_{x,y}(n)\, dy$$

$$\omega^+_{x,y}(n) = \mathbb{P}_{-x,0}\left(\Omega^+_{n-2} \mid \varphi_{n-1} = y,\ \varphi_n = 0 \right)$$

<u>Markov renewal process</u> $(\tau_1, \tau_2, \dots)$

<u>Modulating chain</u> $(J_1, J_2, \dots)$

<u>Perron - Frobenius</u>

$$K^{\varepsilon,(+)}_{x,y}(n) = \varepsilon\, f^{(+)}_{x,y}(n)\, e^{-F^{(+)}(\varepsilon)\, n}\, \frac{u^{(+)}_\varepsilon(y)}{u^{(+)}_\varepsilon(x)}$$

$$\mathbb{P}^{(+)}_\varepsilon\left((\tau_{k+1}, J_{k+1}) = (n,y) \mid (\tau_k, J_k) = (m,x) \right) = K^{\varepsilon,(+)}_{x,y}(n-m)$$

$(T_x, J_x)_{x \in \mathbb{N}}$ is <u>defective</u> (sub Markovian) if $\varepsilon < \varepsilon_c^{(+)}$

$(T_x, J_x)_{x \in \mathbb{N}}$ is <u>proper</u> if $\varepsilon > \varepsilon_c^{(+)}$

we can express $F^{(+)}(\varepsilon)$ in terms of $\delta^{(+)}(\lambda)$

Spectral radius of

$$B^{(+)\lambda}_{x,dy} = \sum_n e^{-\lambda n} F^{(+)}_{x,dy}(n)$$

$$F^{(+)}(\varepsilon) = \begin{cases} \left(\delta^{(+)}\right)^{-1}(1/\varepsilon) & \varepsilon > \varepsilon_c^{(+)} \\ 0 & \varepsilon \le \varepsilon_c^{(+)} \end{cases}$$

ie $\varepsilon_c^{(+)} = \dfrac{1}{\delta^{(+)}(0)}$, use

$f_{x,y}(n) \simeq \dfrac{c}{n^2}$ $\qquad f^{(+)}_{x,y}(n) \simeq \dfrac{c}{n^{2+\varepsilon}}$ $\qquad \varepsilon = 1/2 > 0$